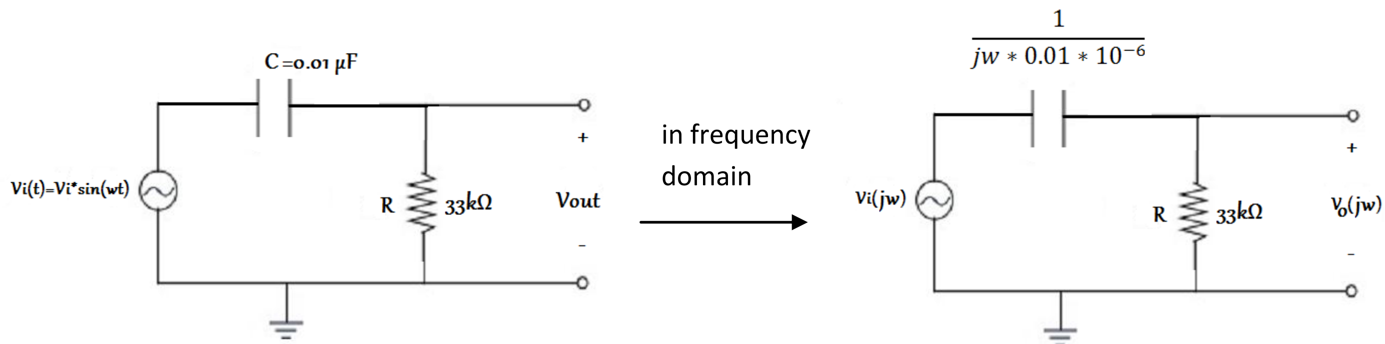
Ece 232 - Advanced Electrical Circuit Analysis Lab Manual 5

Frequency Response



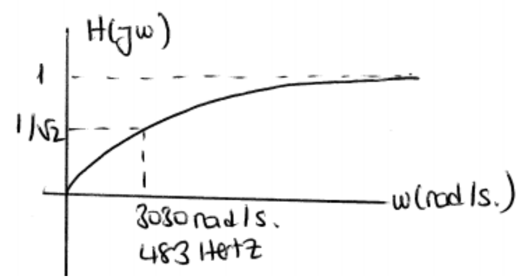
1. $$H(jw)=\frac{V_o(jw)}{V_i(jw)}=\frac{33*10^3}{33*10^3-\frac{10^8}{w}j}=\frac{33*10^3w}{33*10^3w-10^8j}$$

$$(if\ w=0\ \ |H(jw)|=0, if\ w=\infty\ \ |H(jw)|=1):high-pass\ filter$$

<u>Magnitude</u> $(|H(jw)|)_{max}=1$

$$|H(jw)|=\frac{33*10^3w}{\sqrt{(33*10^3w)^2+10^{16}}}$$

For corner frequency (half power frequency)



$$|H(jw_c)|=\frac{1}{\sqrt{2}}|H(jw)|_{max}$$

$$\frac{(33*10^3*w_c)^2}{33^2*10^6*w_c{}^2+10^{16}}=\frac{1}{2}$$

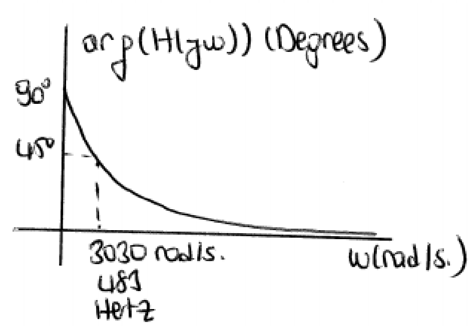
$$w_c{}^2=\frac{10^{16}}{33^2*10^6}$$

$$w_c=\frac{10^5}{33}\cong 3030rad/\sec \qquad f_c=\frac{3030}{2\pi}=483Hertz$$

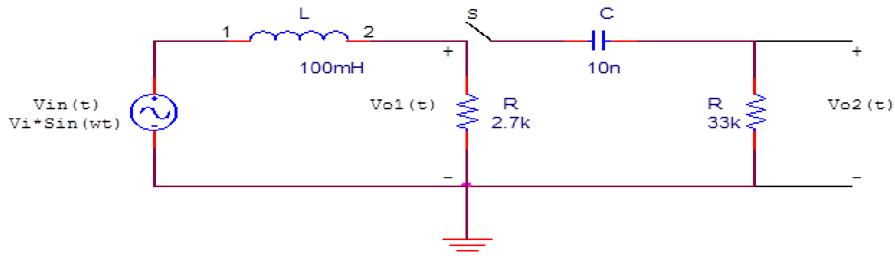
<u>phase</u>

$$H(jw)=\frac{33*10^3w}{33*10^3w-10^8j}=\frac{33*10^3w*(33*10^3w+10^8j)}{33^2*10^6w^2+10^{16}}$$

$$\arg\big(H(jw)\big)=\tan^{-1}\left(\frac{10^8}{33*10^3w}\right)$$

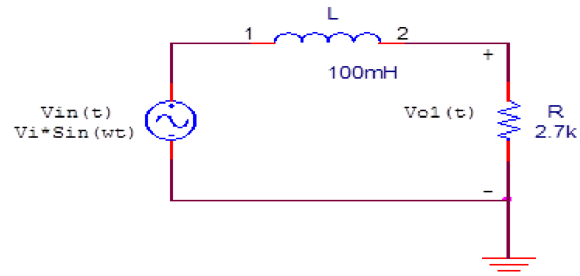


2.



a. S is open

<u>in phasor domain</u>



$$H_1(jw) = \frac{V_{o1}(jw)}{V_i(jw)} = \frac{2700}{2700 + 0.1jw}$$

$$(if\ w = 0\ \ |H(jw)| = 1, if\ w = \infty\ \ |H(jw)| = 0) : low - pass\ filter$$

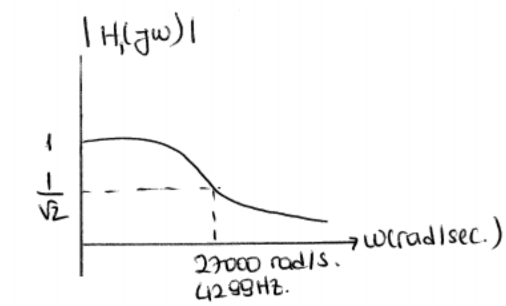
<u>Magnitude</u> $(|H_1(jw)|)_{max} = 1$

$$|H_1(jw)| = \frac{2700}{\sqrt{2700^2 + 0.1^2 w^2}}$$

<u>phase</u>

$$H_1(jw) = \frac{2700 * (2700 - 0.1jw)}{2700^2 + 0.1^2 w^2}$$

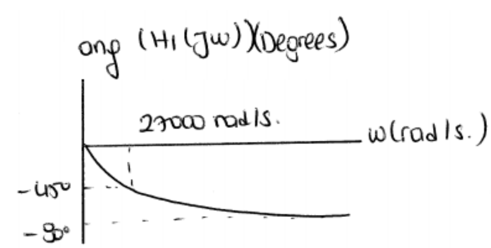
$$\arg\big(H(jw)\big) = \tan^{-1}(\frac{-0.1w}{2700}) = \tan^{-1}(\frac{-w}{27000})$$



<u>For half-power frequency w_c</u>

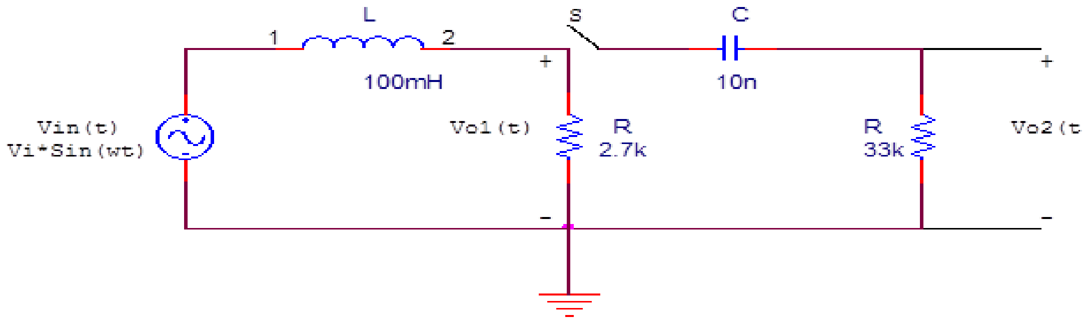
$$|H(jw_c)| = \frac{1}{\sqrt{2}} |H(jw)|_{max}$$

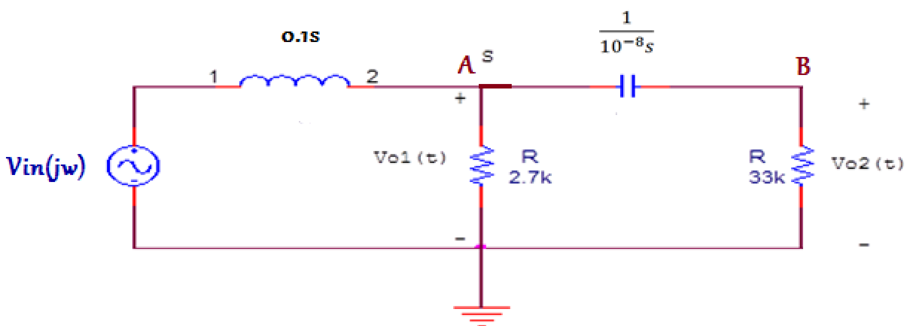
$$\frac{2700^2}{2700^2 * (0.1w_c)^2} = \frac{1}{2}$$



$$w_c = 27000 rad/sec \qquad f_c = \frac{27000}{2\pi} = 4299 Hertz$$

b.





$$\frac{\mathrm{V_i - A}}{0.1\mathrm{s}} = \frac{\mathrm{A}}{2700} + \frac{\mathrm{As}}{10^8 + 33000\mathrm{s}}$$

$$\mathrm{V_i} = \mathrm{A}\left(\frac{27 * 10^9 + 991 * 10^4\mathrm{s} + 357\mathrm{s}^2}{27 * 10^9 + 891 * 10^4\mathrm{s}}\right)$$

$$\frac{\mathrm{A}}{\frac{10^8}{\mathrm{s}} + 33000} = \frac{\mathrm{B}}{33000}$$

$$\mathrm{A} = \mathrm{B}\frac{33\mathrm{s} + 10^5}{33\mathrm{s}}$$

$$\mathrm{V_i} = \mathrm{B}\frac{33\mathrm{s} + 10^5}{33\mathrm{s}}\left(\frac{27 * 10^9 + 991 * 10^4\mathrm{s} + 357\mathrm{s}^2}{27 * 10^9 + 891 * 10^4\mathrm{s}}\right)$$

$$\mathrm{B} = \mathrm{V_{o_2}}$$

$$\mathrm{V_i} = \mathrm{V_{o_2}}\frac{33\mathrm{s} + 10^5}{33\mathrm{s}}\left(\frac{27 * 10^9 + 991 * 10^4\mathrm{s} + 357\mathrm{s}^2}{27 * 10^4(33\mathrm{s} + 10^5)}\right)$$

$$\frac{\mathrm{V_{o_2}}}{\mathrm{V_i}} = H_2(s) = \frac{33 * 27 * 10^4 * \mathrm{s}}{27 * 10^9 + 991 * 10^4\mathrm{s} + 357\mathrm{s}^2}$$

$$H_2(jw) = \frac{891 * 10^4 * \mathrm{jw}}{27 * 10^9 + 991 * 10^4\mathrm{jw} - 357\mathrm{w}^2}$$

Magnitude response

$$|H_2(jw)| = \frac{891 * 10^4 * \mathrm{w}}{\sqrt{991^2 * 10^8\mathrm{w}^2 + (27 * 10^9 - 357\mathrm{w}^2)^2}}$$

Phase response

$$H_2(jw) = \frac{891 * 10^4 * \mathrm{jw}}{27 * 10^9 + 991 * 10^4\mathrm{jw} - 357\mathrm{w}^2}$$

by multiplying by conjugate

$$Arg\big(H_2(jw)\big) = \tan^{-1}\frac{27 * 10^9\mathrm{w} - 357\mathrm{w}^3}{991 * 10^4 * w^2} = \tan^{-1}\frac{27 * 10^9 - 357\mathrm{w}^2}{991 * 10^4 * w}$$

For resonant angular frequency the imaginary part of the response must be zero

$$H_2(jw) = \frac{891 * 10^4 * [(27 * 10^9\mathrm{w} - 357\mathrm{w}^3)\mathrm{j} + 991 * 10^4\mathrm{w}^2]}{(27 * 10^9 - 357\mathrm{w}^2)^2 + (991 * 10^4\mathrm{w})^2}$$

$$27 * 10^9\mathrm{w} - 357\mathrm{w}^3 = 0$$

$$\mathrm{w} = 0 \quad \text{or}$$

$$w^2 = \frac{27 * 10^9}{357}$$

$$w = \sqrt{\frac{27 * 10^9}{357}} = 0.8697 * 10^4 = 8697\frac{\mathrm{rad}}{\mathrm{sec}} = 1385\mathrm{Hertz}$$

The magnitude at the resonant angular frequency

$$H_2(jw) = \frac{891 * 10^4 * \left[\left(\frac{27 * 10^9}{357}\right) * 991 * 10^4\right]}{\left(\frac{27 * 10^9}{357}\right) * (991 * 10^4)^2} = \frac{891}{991} = 0.899$$

When does $|H_2(jw)|$ reaches its maximum value

$$\frac{d}{dw}|H_2(jw)| = \frac{891 * 10^4 * [-2 * 127449\mathrm{w}^4 + 2 * 729 * 10^{18}]}{2(127449\mathrm{w}^4 + 789301 * 10^8\mathrm{w}^2 + 729 * 10^{16})} = 0$$

$$\mathrm{w}^4 = 2 * \frac{729 * 10^{18}}{2 * 127449}$$

$$\mathrm{w}^2 = \frac{27 * 10^9}{357}$$

$$w \cong \frac{8697rad}{sec} = 1385Hertz$$

These results indicates that the max value for $|H_2(jw)|$, obtained when the circuit is operating in the resonant frequency. $arg(|H_2(jw)|)_{max} = w_{resonant-frequency}$

$|H_2(jw)| = 1$ This value is not possible since $(|H_2(jw)|)_{max} = 0.89$

To find the corner frequencies (half-power frequencies) find the frequencies where phase= $\mp 45^o$

April 2012

First corner frequency

$$\tan^{-1}\frac{27*10^9-357w^2}{991*10^4*w}=45^o$$

$$1=\frac{27*10^9-357w^2}{991*10^4*w}$$

$$0=357w^2+991*10^4*w-27*10^9$$

$$w_{1,2}=\frac{-991*10^4\mp\sqrt{(991*10^4)^2+4*357*27*10^9}}{2*357}=0.2499*10^4=\frac{2499rad}{\sec}=398Hertz$$

Second corner frequency

$$\tan^{-1}\frac{27*10^9-357w^2}{991*10^4*w}=-45^o$$

$$-1=\frac{27*10^9-357w^2}{991*10^4*w}$$

$$0=357w^2-991*10^4*w-27*10^9$$

$$w_{1,2}=\frac{991*10^4\mp\sqrt{(991*10^4)^2+4*357*27*10^9}}{2*357}=30259red/sec=4818Hertz$$

$|H_2(jw)|$ at corner frequencies

$$H_2(jw)=\frac{891*10^4*991*10^4w^2\left[\frac{(27*10^9w-357w^3)}{991*10^4w^2}j+1\right]}{(991*10^4w)^2\left(1+\frac{27*10^9-357w^2}{991*10^4w}\right)^2}$$

$$at\quad w_{c1}\qquad \frac{27*10^9-357w^2}{991*10^4*w}=1$$

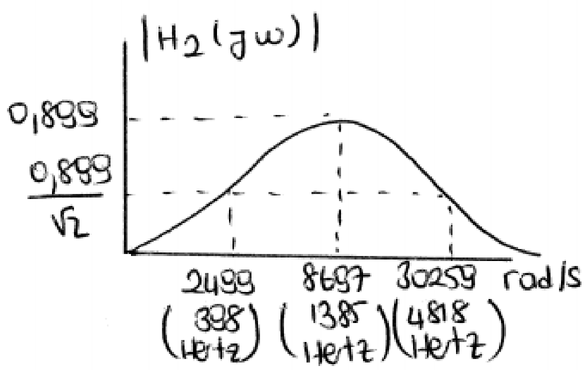
$$H_2(w_{c1})=\frac{891}{991}*\frac{1+j}{1+1^2}=\frac{891}{991}*\frac{1+j}{2}$$

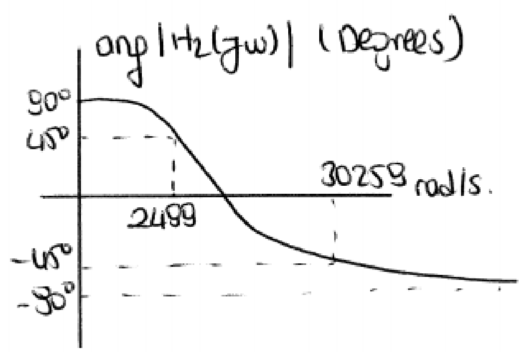
$$|H_2(w_{c1})|=\frac{891}{991}*\frac{1}{\sqrt{2}}=\frac{0.899}{\sqrt{2}}$$

$$at\quad w_{c2}\qquad \frac{27*10^9-357w^2}{991*10^4*w}=-1$$

$$H_2(w_{c2})=\frac{891}{991}*\frac{1-j}{1+1^2}=\frac{891}{991}*\frac{1-j}{2}$$

$$|H_2(w_{c2})|=\frac{891}{991}*\frac{1}{\sqrt{2}}=\frac{0.899}{\sqrt{2}}$$





$$\Delta w = w_{c2} - w_{c1} = 27760 rad/sec$$

$$|w_1 - w_2| = bandwidth$$

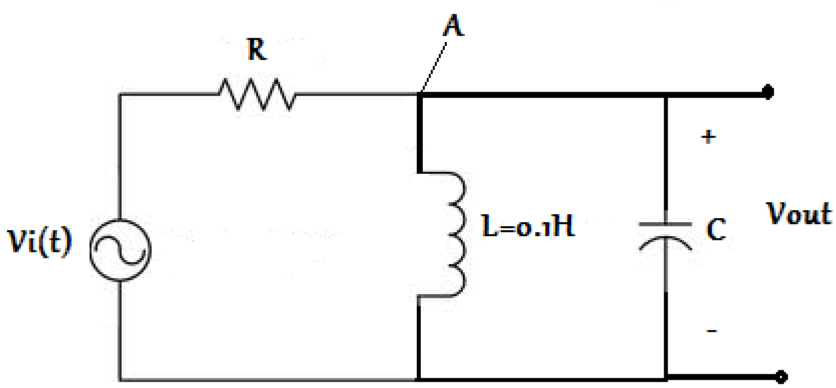
$$H(s) * H_1(s) = \frac{(33 * 10^3)}{\left(33s * 10^3 + \frac{10^8}{s}\right)} * \frac{2700}{2700 + 0.1s}$$

$$H(s) * H_1(s) = \frac{(5 * 33 * 27 * 10^5)}{(33s + 10^5)10^3(27000 + s)10^{-1}} \cong \frac{249575}{(s + 24696)(s + 3063)}$$

We can see that the gain the zeros and poles of two systems are so close to each other as seen ………….. allows.

3. L=0.1H, $w_o = 10krad/sec, V_i(t) = Vsin(wt)$

$$f_o = 1592Hertz$$



$$\frac{\mathrm{V_i - A}}{\mathrm{R}} = \frac{\mathrm{A}}{\mathrm{jw * 0.1}} + \frac{\mathrm{A}}{\frac{1}{\mathrm{jwC}}}$$

$$\frac{\mathrm{A}}{\mathrm{V_i}} = \frac{\mathrm{w}}{\mathrm{w + j(w^2CR - 10R)}} = \frac{\mathrm{V_o(jw)}}{\mathrm{V_i(jw)}} = \mathrm{H(jw)}$$

a. $R = 3.3k\Omega$

To find the resonant angular frequency

$$\mathrm{w^2CR - 10R = 0}$$

$$C = \frac{10}{{w_o}^2} = \frac{10}{10^8} = 0.1\mu F$$

Value of C independent of R.

To find half-power frequency

$$\frac{w_c^2}{w_c^2 + (w_c^2 CR - 10R)^2} = \frac{1}{2}$$

$$w_c^4 - \left(2 + \frac{1}{3.3^2}\right) 10^8 w_c^2 + 10^{16} = 0$$

$w_{c1} = 8599 rad/\sec$ $f_1 = 1369 Hertz$

$w_{c2} = 11625 rad/\sec$ $f_2 = 1851 Hertz$

$$\Delta w = w_{c2} - w_{c1} = 3029 rad/sec$$

$$Q = \frac{w_c}{\Delta w} = \frac{10000}{3029} = 3.3$$

a. $R = 10k\Omega$, L=0.1H, $w_o = 10krad/sec, f_o = 1592Hertz$

$$C = 0.1\mu F$$

Since it is independent of R.

To find half-power frequency

$$\frac{w_c^2}{w_c^2 + (w_c^2 CR - 10R)^2} = \frac{1}{2}$$

$$w_c^4 C^2 R^2 - 20 w_c^2 CR^2 - w_c^2 + 100R^2 = 0$$

$w_{c1} = 10510 rad/\sec$ $f_1 = 1673 Hertz$

$w_{c2} = 9517 rad/\sec$ $f_2 = 1514 Hertz$

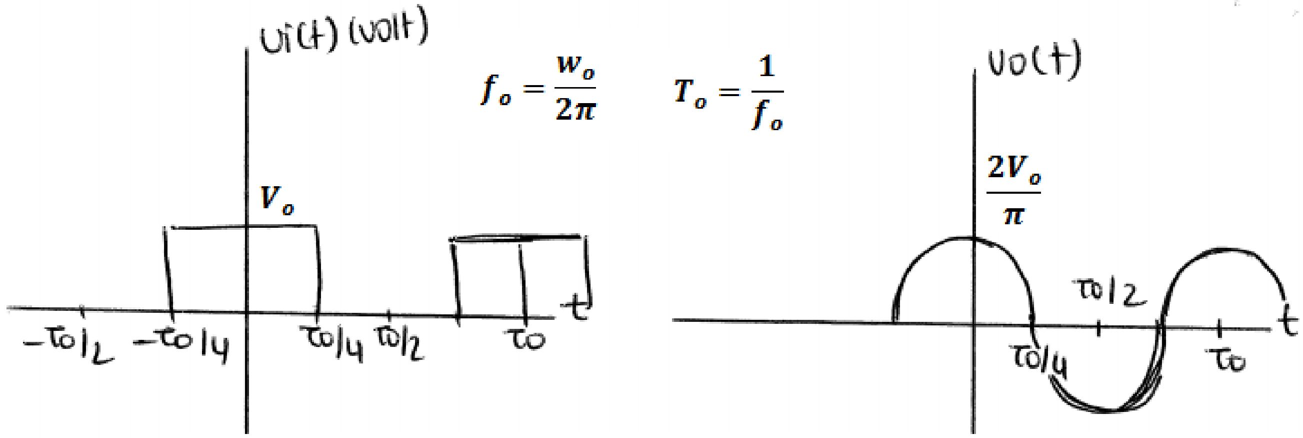
$$\Delta w = w_{c2} - w_{c1} = 998 rad/sec$$

$$Q = \frac{w_c}{\Delta w} = \frac{10000}{998} \cong 10$$

$$H(jw) = \frac{w}{w + j(w^2 10^{-7} 10^4 - 10 * 10^4)}$$

$$|H(jw)| = \frac{w}{\sqrt{w^2 + (w^2 10^{-3} - 10^5)^2}}$$

$$\arg(H(jw)) = \tan^{-1}\left(\frac{-w^2 10^{-3} - 10^5}{w}\right)$$



$$f_o = \frac{w_o}{2\pi} \qquad T_o = \frac{1}{f_o}$$

$$V_i(t) = V_o\left[\frac{1}{2} + \frac{2}{\pi}\cos(w_o t) - \frac{2}{3\pi}\cos(3w_o t) + \frac{2}{5\pi}\cos(5w_o t) - \cdots\right]$$

Since the filter is a band-pass filter with critical frequency w_o only the components of $V_i(t)$ with frequency w_o passes. DC components and other frequencies dies out at the output.

4. In Preliminary work part 1, the filter is a high-pass filter.

In Preliminary work part 2a, the filter is a low-pass filter.

In Preliminary work part 2b, the filter is a band-pass filter.