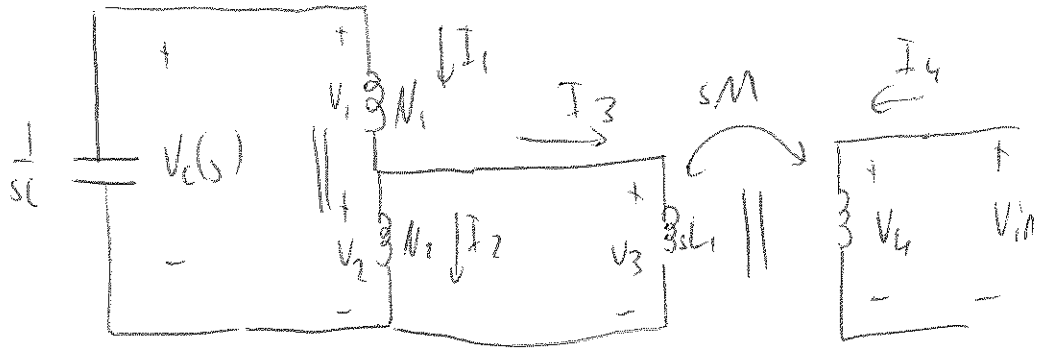
2013-2014

ECE 232 Homework 3

Q-1 In Laplace domain



in Laplace domain →

$$V_3 = sL_1 I_3 + sM I_4$$
$$V_4 = sM I_3 + sL_2 I_4$$

$$V_1 = \frac{N_1}{N_2} V_2 \qquad I_1 = -\frac{N_2}{N_1} I_2$$

$$V_c = -I_1 \frac{1}{sC} \qquad I_3 = I_1 - I_2 = I_1 - \left(-\frac{N_1}{N_2}\right) I_1 = \left(1 + \frac{N_1}{N_2}\right) I_1$$

$$V_{in} = V_4 = sM I_3 + sL_2 I_4$$

$$\boxed{V_{in} = V_4 = sM\left(1 + \frac{N_1}{N_2}\right) I_1 + sL_2 I_4}$$

$$V_3 = V_2 = V_c - V_1$$

$$V_2 = V_c - \frac{N_1}{N_2} V_2$$

$$\left(1 + \frac{N_1}{N_2}\right) V_2 = V_c$$

$$V_3 = sL_1 I_3 + sM I_4$$

$$\frac{N_2}{N_1 + N_2} V_c = sL_1 \left(\frac{N_2 + N_1}{N_2}\right) I_1 + sM I_4$$

$$\frac{\frac{N_1 + N_2}{N_2} V_2 = V_c}{\frac{N_1 + N_2}{N_2}}$$

$$\boxed{V_2 = V_3 = \frac{N_2}{N_1 + N_2} V_c}$$

Q-1 (continue)

$$\frac{V_{in} - sM\left(1 + \frac{N_1}{N_2}\right) I_1}{sL_2} = \frac{\frac{N_2}{N_1 N_2} V_c - sL_1 \left(\frac{N_2 + N_1}{N_2}\right) I_1}{sM} = I_4$$

$$sMV_{in} - s^2M^2\left(\frac{N_1+N_2}{N_2}\right) I_1 = \frac{N_2 sL_2}{N_1+N_2} V_c - s^2 L_1 L_2 \left(\frac{N_2+N_1}{N_2}\right) I_1$$

$$sMV_{in} = \frac{sL_2 N_2}{N_1+N_2} V_c + \left(s^2M^2 - s^2L_1L_2\right)\left(\frac{N_1+N_2}{N_2}\right) I_1$$

$$\boxed{I_1 = -V_c sC}$$

$$sMV_{in} = \frac{sL_2N_2}{N_1+N_2} V_c + \left(s^2L_1L_2 - s^2M^2\right) sC \left(\frac{N_1+N_2}{N_2}\right) V_c$$

$$sMV_{in} = \left[\frac{sL_2N_2^2}{(N_1+N_2)N_2} + \frac{\left(s^2L_1L_2 - s^2M^2\right)(N_1^2+N_2^2)\, sC}{(N_1+N_2)N_2}\right] V_c$$

$$\frac{V_c}{V_{in}} = \frac{sM N_2 (N_1+N_2)}{s\left[\left(s^2L_1L_2 - s^2M^2\right)(N_1^2+N_2^2)C + N_2^2 L_2\right]}$$

$$\frac{V_c}{V_{in}} = \frac{M(N_2)(N_1+N_2)}{s^2\left(L_1L_2 - M^2\right)(N_1^2+N_2^2)C + N_2^2 L_2}$$

Q-2

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

* $A = \left.\frac{V_1}{V_2}\right|_{-I_2=0}$

$I_2 = 0 = g_{21} V_1 + g_{22} V_2$

$\hookrightarrow \frac{V_1}{V_2} = -\frac{g_{22}}{g_{21}} = A$ //

* $C = \left.\frac{I_1}{V_2}\right|_{-I_2=0}$

$I_1 = g_{11} V_1 + g_{12} V_2$

$\frac{I_1}{V_2} = g_{11}\left(\frac{V_1}{V_2}\right) + g_{12}\frac{V_2}{V_2}$

$\frac{I_1}{V_2} = g_{11}\left(-\frac{g_{22}}{g_{21}}\right) + g_{12}$

$\frac{I_1}{V_2} = C = g_{12} - \frac{g_{11} g_{22}}{g_{21}}$ //

* $B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$

$I_1 = g_{11} V_1 + g_{12} V_2$

$I_2 = g_{21} V_1 + g_{22} V_2$

if $V_2 = 0$ $\rightarrow$ $I_1 = g_{11} V_1$, $I_2 = g_{21} V_1$

$B = -\frac{V_1}{I_2} = -\frac{1}{g_{21}}$ //

* $D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = -\frac{g_{11}}{g_{21}}$ //