ELE 232 Homework II

2013-2014

Q-1

$$H(j\omega) = \frac{2}{1+10^{-2}(j\omega)^2+10^{-1}j\omega} = \frac{2}{(1-10^{-2}\omega^2)+10^{-1}j\omega}$$

(a)

$$|H(j\omega)| = \frac{2}{\sqrt{\left(1-\frac{\omega^2}{100}\right)^2+\left(\frac{\omega}{10}\right)^2}}$$

$$|H(j\omega)|\Big|_{\omega=0} = 2 \qquad \lim_{\omega\to\infty}|H(j\omega)| = 0 \qquad |H(j\omega)|\Big|_{\omega=10} = 2$$

$$\frac{d|H(j\omega)|}{d\omega} = 2\left[\frac{0 - 1\times\left[2\left(1-\frac{\omega^2}{100}\right)\left(\frac{-2\omega}{100}\right)+2\left(\frac{\omega}{10}\right)\frac{1}{10}\right]\left(\left(1-\frac{\omega^2}{100}\right)^2+\left(\frac{\omega}{10}\right)^2\right)^{-\frac{1}{2}}}{\left(1-\frac{\omega^2}{100}\right)^2+\left(\frac{\omega}{10}\right)^2}\right]$$

$$= -2\,\frac{2\left(1-\frac{\omega^2}{100}\right)\left(\frac{-2\omega}{100}\right)+2\left(\frac{\omega}{10}\right)\frac{1}{10}}{\left[\left(1-\frac{\omega^2}{100}\right)^2+\left(\frac{\omega}{10}\right)^2\right]^{\frac{3}{2}}}$$

$$= -2\,\frac{\frac{1}{50}\left[\left(1-\frac{\omega^2}{100}\right)(-2\omega)+\omega\right]}{\left[\left(1-\frac{\omega^2}{100}\right)^2+\left(\frac{\omega}{10}\right)^2\right]^{\frac{3}{2}}} = -2\,\frac{\frac{1}{50}\left[\omega\left(-2+\frac{\omega^2}{50}+\omega\right)\right]}{\left[\left(1-\frac{\omega^2}{100}\right)^2+\left(\frac{\omega}{10}\right)^2\right]^{\frac{3}{2}}}$$

if $\frac{d|H(j\omega)|}{d\omega} = 0 \Rightarrow \omega = 0$ or $\left(-2+\frac{\omega^2}{50}+\omega\right) = 0 \Rightarrow \omega^2+50\omega-100=0$

$$\omega = \frac{-50\mp\sqrt{2500+400}}{2}$$

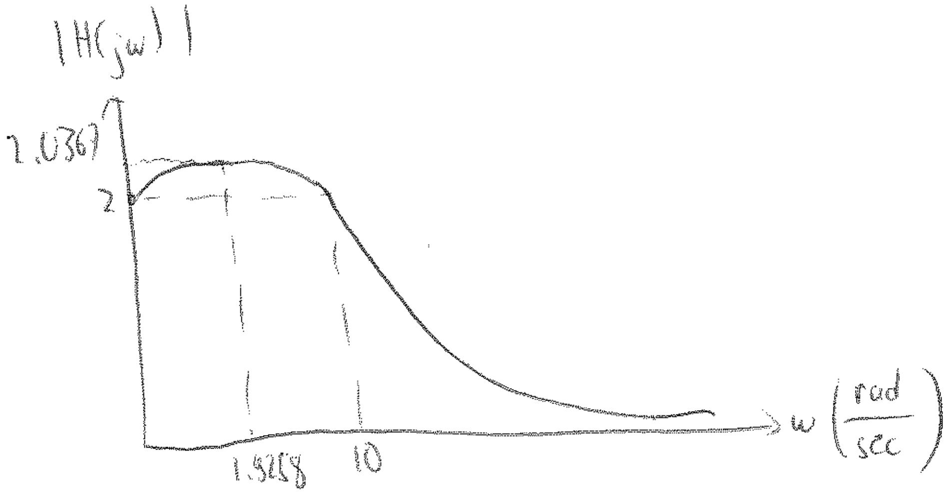
$$\omega_{1,2} = \frac{-50\mp\sqrt{2900}}{2} = -25\mp 5\sqrt{29} \begin{cases} \omega_1 = 5\sqrt{29}-25 > 0 \\ \omega_2 = -5\sqrt{29}-25 < 0 \end{cases}$$

$*$

Q-1 (continue)

when $w = 5\sqrt{29} - 25 = 1.92582$ $|H(jw)|$ is maximized

$$\left. |H(jw)| \right|_{w = 5\sqrt{29} - 25} \cong 2.0367$$



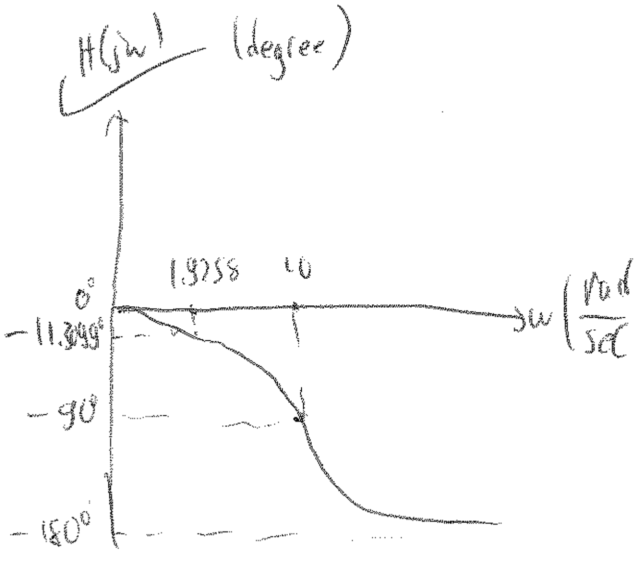
(b) $\angle H(jw) = \tan^{-1} \dfrac{\frac{w}{10}}{1 - \frac{w^2}{100}} = \dfrac{N(w)}{D(w)}$ $\left. \angle H(jw) \right|_{w=0} = -\tan^{-1} \dfrac{0}{1} = 0°$

$\lim_{w \to \infty} \angle H(jw) = -\tan^{-1} \dfrac{\infty}{-\infty}$ however $\deg N(w) = 1 < \deg D(w) = 2$

$= -180°$

$\lim_{w \to 10^-} \angle H(jw) = -90$ $\lim_{w \to 10^+} \angle H(jw) = -90°$

$\lim_{w \to 5\sqrt{29} - 25} \angle H(jw) = -11.3099°$



Q-2-

$$|H(j\omega)| = \frac{100-\omega^2}{\sqrt{(100-\omega^2)^2 + (15\omega)^2}} = \frac{|100-\omega^2|}{\sqrt{\omega^4 + 25\omega^2 + 10^4}} = \frac{|100-\omega^2|}{D(\omega)}$$

(a)

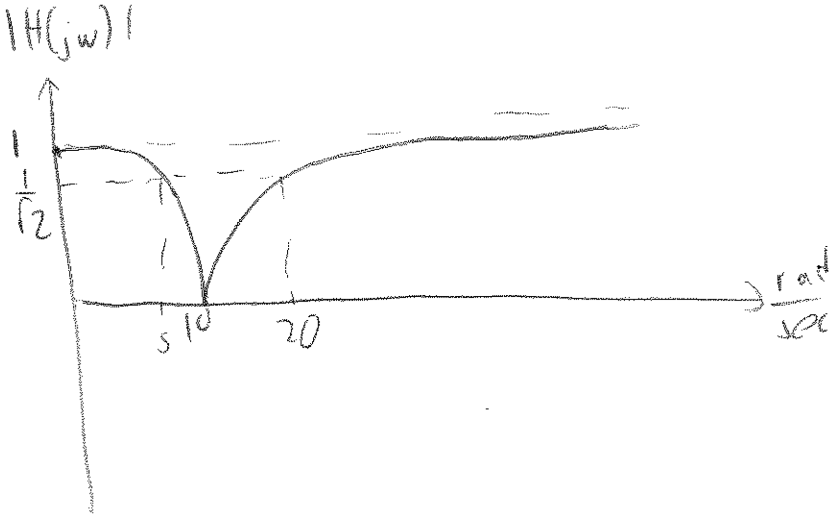
$$|H(j\omega)|\Big|_{\omega=0} = 1 \qquad |H(j\omega)|\Big|_{\omega=10} = 0$$

$$\lim_{\omega\to\infty} |H(j\omega)| = 1$$

$$\frac{d|H(j\omega)|}{d\omega} = \begin{cases} \dfrac{-\left[2\omega D(\omega) - \dfrac{dD(\omega)}{d\omega}(100-\omega^2)\right]}{D^2(\omega)} & 0<\omega<10 \;\rightarrow \text{negative} \\[2ex] \dfrac{2\omega D(\omega) + \dfrac{dD(\omega)}{d\omega}(\omega^2-100)}{D^2(\omega)} & 10<\omega \;\rightarrow \text{positive} \end{cases}$$

~~$d|H(j\omega)|$~~ $|H(j\omega)| \rightarrow$ decreasing when $0<\omega<10$

$|H(j\omega)| \rightarrow$ increasing when $10<\omega$



$$\boxed{\text{if } \omega = 5 \quad |H(j\omega)| = \frac{1}{\sqrt{2}}}$$

$$\boxed{\text{if } \omega = 20 \quad |H(j\omega)| = \frac{1}{\sqrt{2}}}$$

(Q-2) continue

(b) $\angle H(jw) = \begin{cases} 0^\circ - \tan^{-1} \dfrac{15w}{100-w^2} & 0 < w < 10 \\ 180 - \tan^{-1} \dfrac{15w}{100-w^2} & 10 < w \end{cases}$

$\lim\limits_{w \to 10^-} \angle H(jw) = -90^\circ \qquad \lim\limits_{w \to 10^+} \angle H(jw) = 90^\circ$

if $\dfrac{15w}{100-w^2} = 1 \qquad 15w = 100 - w^2 \qquad w^2 + 15w - 100 = 0$

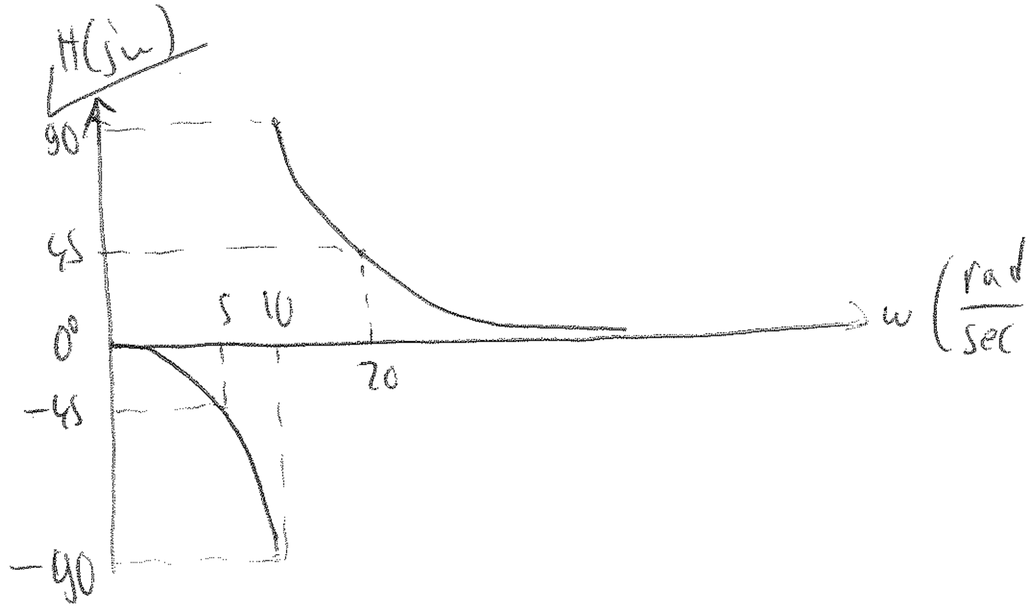
$w_{1,2} = \dfrac{-15 \mp \sqrt{225+400}}{2} = \dfrac{-15 \mp 25}{2} = -20, 5 \ \dfrac{rad}{sec}$

$$\boxed{\text{if } w = 5 \quad \angle H(jw) = 0 - \tan^{-1} \frac{1}{1} = -45^\circ}$$

if $\dfrac{15w}{100-w^2} = -1 \qquad 15w = -100 + w^2 \qquad w^2 - 15w - 100 = 0$

$w_{1,2} = \dfrac{15 \mp \sqrt{225+400}}{2} = 20, -5 \ \dfrac{rad}{sec}$

if $w = 20 \quad \angle H(jw) = 180 - \tan^{-1} \dfrac{300}{-300} = 180 - 135 = 45^\circ$

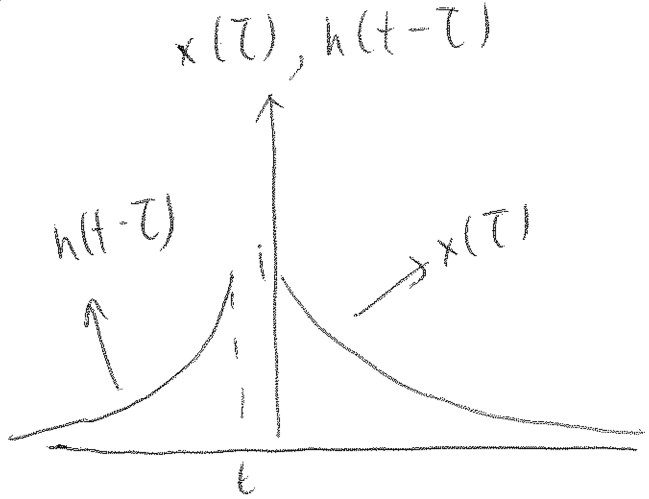


Q-3 $h(t) = e^{-t}u(t)$ $x(t) = e^{-t}u(t)$

$y(t) = \int_{-\infty}^{t} x(\tau)\,h(t-\tau)\,d\tau$

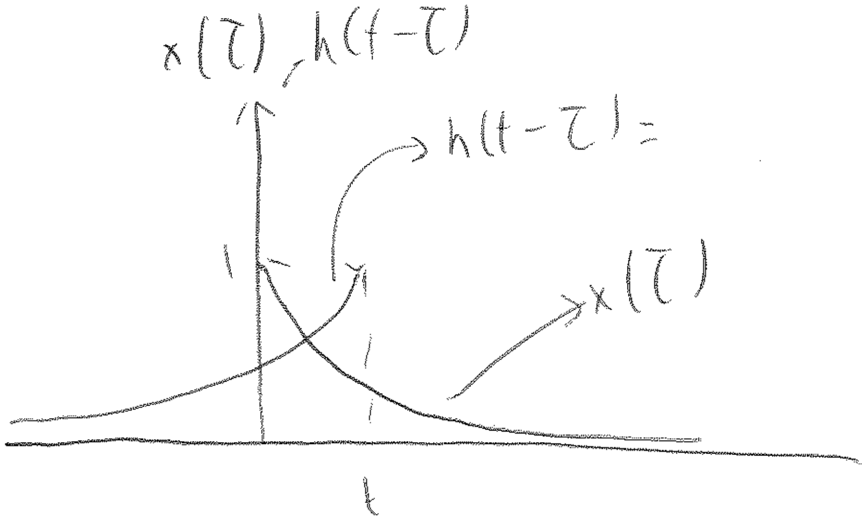
$x(\tau) = e^{-\tau}u(\tau)$, $h(t-\tau) = e^{-(t-\tau)}u(t-\tau)$

if $t<0$



$y(t) = \int_{-\infty}^{t} x(\tau)h(t-\tau)d\tau = 0$

if $t>0$



$y(t) = \int_{-\infty}^{t} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{0} 0 + \int_{0}^{t} e^{-\tau}\underbrace{u(\tau)}_{1}\, e^{-(t-\tau)}\underbrace{u(t-\tau)}_{1}d\tau$

$= \int_{0}^{t} e^{-t}e^{-\tau}e^{\tau}d\tau = e^{-t}\int_{0}^{t} 1\, d\tau$

$= e^{-t}\,\tau\Big|_{0}^{t}$

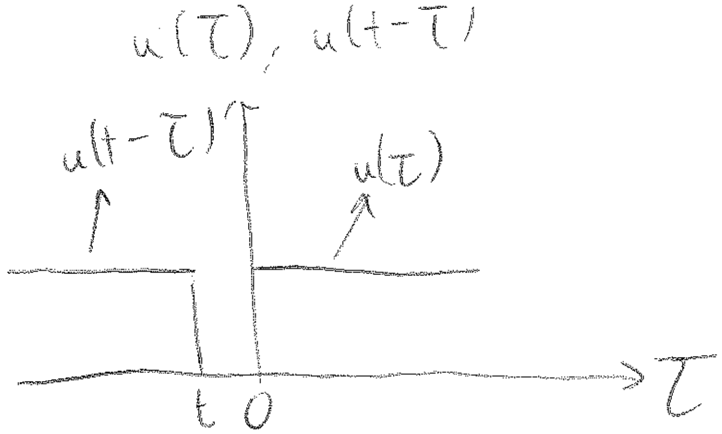
$= te^{-t}$ //

Q-3 continue

hence $y(t) = \begin{cases} 0 & t<0 \\ te^{-t} & t>0 \end{cases}$

Q-4- $y(t) = \int_{-\infty}^{t} u(\tau)\, u(t-\tau)\, d\tau$

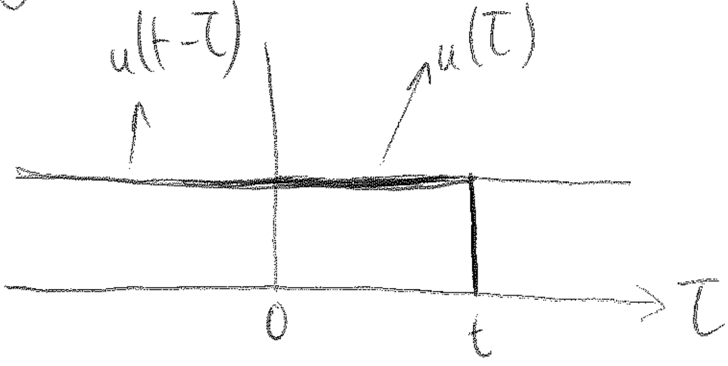
$t<0$



$y(t) = \int_{-\infty}^{t} u(\tau)\, u(t-\tau)\, d\tau$

$= 0$

$t>0$



$y(t) = \int_{-\infty}^{t} u(\tau)\, u(t-\tau)\, d\tau$

$= \int_{-\infty}^{0} 0\, d\tau + \int_{0}^{t} \underbrace{u(\tau)}_{1} \underbrace{u(t-\tau)}_{1} d\tau$

$= \int_{0}^{t} 1\, d\tau = \tau \Big|_0^t = t$

$y(t) = \begin{cases} 0 & t<0 \\ t & t>0 \end{cases} = r(t) = \text{ramp function}$