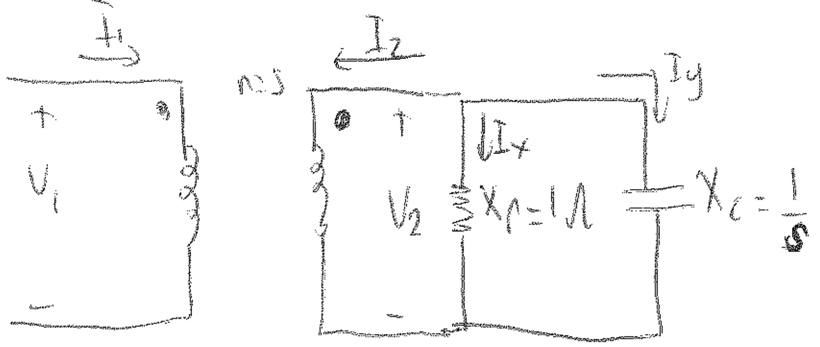
ECE 232 - Example Questions

Q-1

$V_1 = 5V_2 \qquad I_2 = -5I_1$



$V_1 = 5V_2 \qquad I_x = \frac{V_2}{X_r} \qquad I_y = \frac{V_2}{X_c} \qquad I_x + I_y = -I_2$

$I_x = \frac{V_2}{1} \qquad I_y = \frac{V_2}{\frac{1}{s}} \qquad \frac{V_2}{1} + \frac{V_2}{\frac{1}{s}} = -I_2$

$\boxed{V_2[1+s] = -I_2}$

$\boxed{V_2 = \frac{-I_2}{1+s}}$

$V_1 = 5V_2 = 5\,\frac{-I_2}{1+s}$

$\boxed{I_2 = -5I_1}$

$V_1 = 5\,\frac{-I_2}{1+s}$

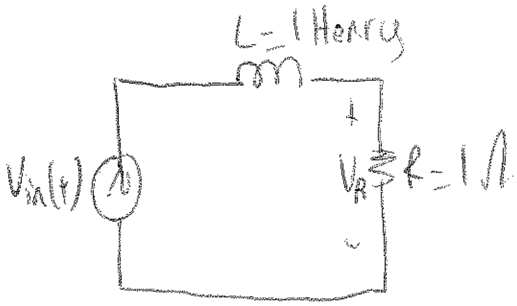
$\downarrow$

$V_1 = 5\,\frac{(-)(-5I_1)}{1+s} \qquad V_1 = \frac{25I_1}{1+s}$

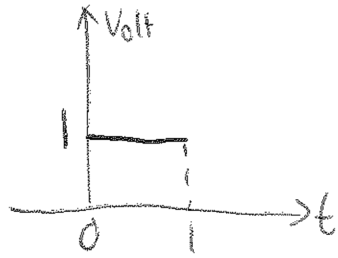
$Z_{in} = \frac{V_1}{I_1} = \frac{25}{1+s}$ //

$u(t) \rightarrow$ unit step input

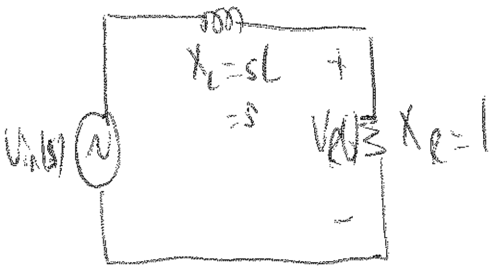
Q-2 -



$$V_{in}(t) = u(t) - u(t-1) = \begin{cases} 1, & 0<t<1 \\ 0, & \text{otherwise} \end{cases}$$



(a) The impulse respons (in Laplace domain)



$$V_R(s) = \frac{1}{1+s} V_{in}(s)$$

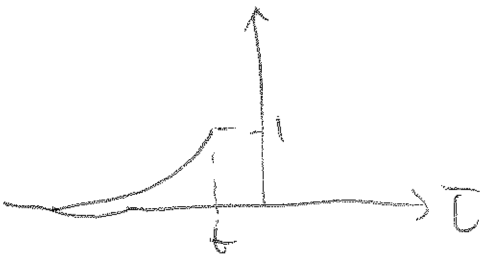
$$\frac{V_R(s)}{V_{in}(s)} = H(s) = \frac{1}{1+s} \rightarrow \text{transfer function}$$

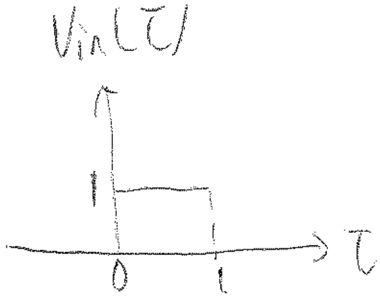
$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{1+s}\right\} = e^{-t} u(t)$$

(b) Convolution integral

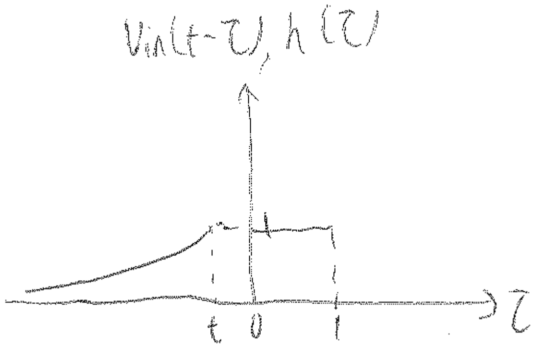
$$V_R(t) = \int_{-\infty}^{t} V_{in}(t-\tau)\, h(\tau)\, d\tau = \int_{-\infty}^{t} V_{in}(\tau)\, h(t-\tau)\, d\tau$$

$$h(t-\tau) = e^{-(t-\tau)}\, u(t-\tau) \quad \left\{ \begin{array}{l} t \text{ parameter} \\ \tau \text{ variable} \end{array} \right\}$$





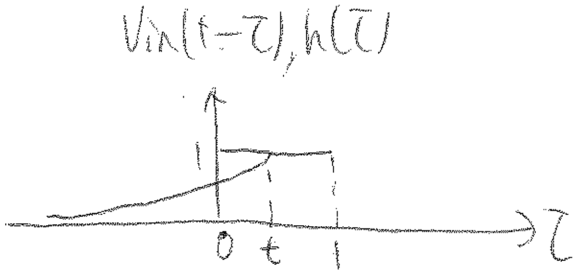
if $t<0$



$$V_R(t) = \int_{-\infty}^{t} V_{in}(t-\tau)\,h(\tau)\,d\tau = \int_{-\infty}^{t} V_{in}(\tau)\,h(t-\tau)\,d\tau$$

$$= 0$$

if $0 \le t < 1$



$$V_R(t) = \int_{-\infty}^{t} V_{in}(\tau)\,h(t-\tau)\,d\tau$$

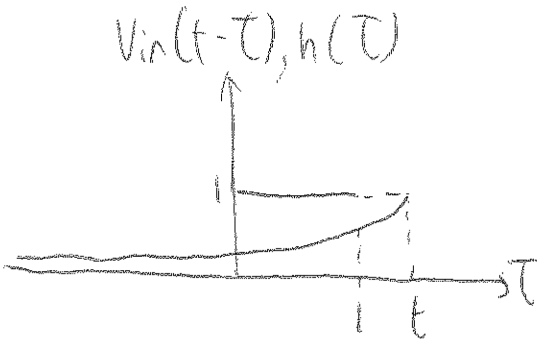
$$= \int_{-\infty}^{0} 0\,d\tau + \int_{0}^{t} e^{-(t-\tau)}\cdot 1\,d\tau$$

$$= \int_{0}^{t} e^{-(t-\tau)}\,d\tau = \underbrace{e^{-t}}_{\text{t is just a variable}} \int_{0}^{t} e^{\tau}\,d\tau = e^{-t}\left[e^{\tau}\Big|_0^t\right]$$

$$= e^{-t}\left[\left[e^{t}-1\right]\right]$$

$$= 1 - e^{-t}$$

if $1 \le t$



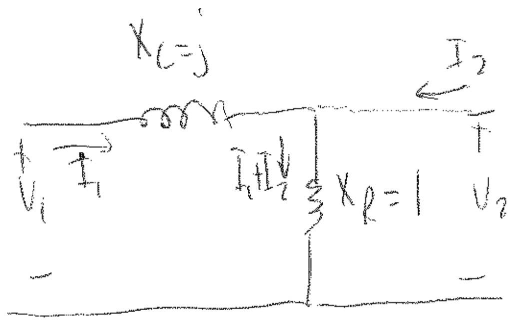
$$V_R(t) = \int_{-\infty}^{t} V_{in}(\tau)\,h(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{0} 0\,d\tau + \int_{0}^{1} e^{-(t-\tau)}\cdot 1\,d\tau + \int_{1}^{t} 0\,d\tau$$

$$= \int_{0}^{1} e^{-(t-\tau)}\,d\tau = e^{-t}\int_{0}^{1} e^{\tau}\,d\tau$$

$$= e^{-t}\left[e^{\tau}\Big|_0^1\right] = e^{-t}\left[e^{1}-1\right]$$

Q-3



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$V_1 = j(I_1) + 1(I_1 + I_2)$ $\qquad$ $V_2 = 1(I_1 + I_2)$

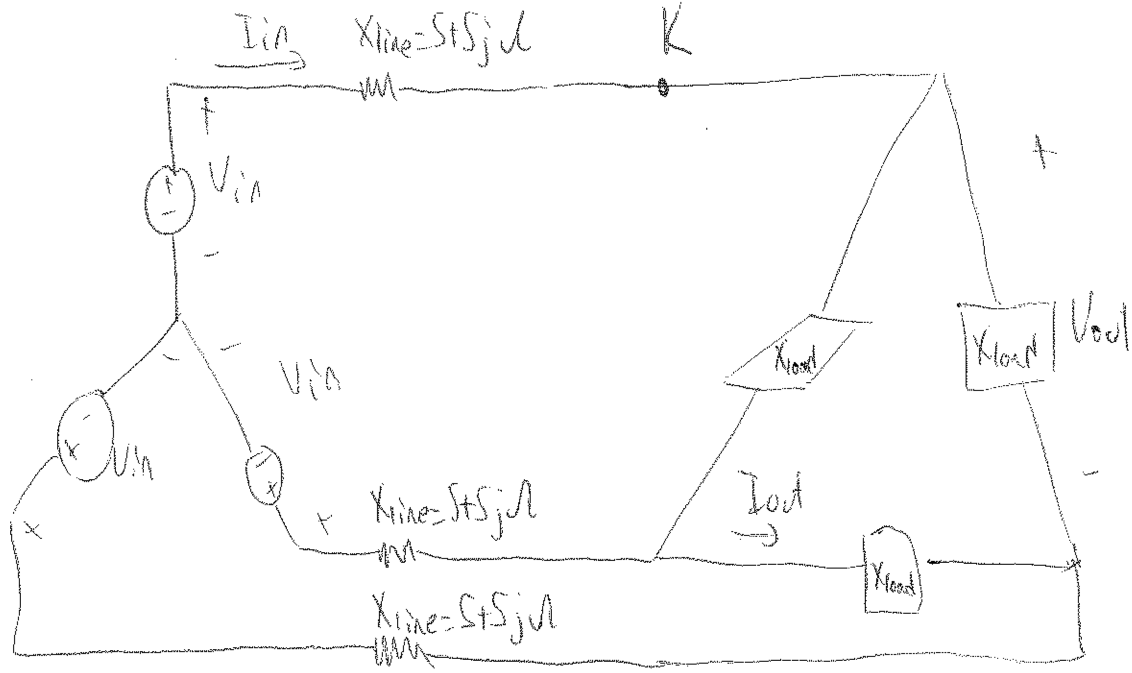
$V_1 = (j+1)I_1 + I_2$ $\qquad$ $V_2 = I_1 + I_2$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j+1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$A = j+1$ $\quad$ $B = 1$

$C = 1$ $\quad$ $D = 1$

Q-4



$P_T = 100$ Watt

$Q_T = -100$ VAR

$|S_T| = \sqrt{P_T^2 + Q_T^2} = 100\sqrt{2}$ VA

complex power

$|S_T| = |V_{in}||I_{in}|\sqrt{3} \Rightarrow 100\sqrt{2} = 100\sqrt{3}\,|I_{in}| \qquad |I_{in}| = \frac{\sqrt{2}}{\sqrt{3}}$ Amper

$P_{Line} = 3\,Re\{X_{line}\}\,|I_{in}|^2 = 3 \times 5 \times \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 = \frac{3 \times 5 \times 2}{3} = 10$ Watt

$Q_{Line} = 3\,Im\{X_{line}\}\,|I_{in}|^2 = 3 \times 5 \times \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 = 10$ VAR

average power at K $\leftarrow P_K = P_T - P_{Line} = 100 - 10 = 90$ Watt

reactive power at K $\leftarrow Q_K = Q_T - Q_{Line} = -100 - 10 = -110$ Watt

$|S_K| = \sqrt{P_K^2 + Q_K^2}$

$= \sqrt{90^2 + (-110)^2}$

$= \sqrt{(81 + 121)\,10^2}$

$= \sqrt{222}\;10$ VA

$$|S_K| = |S_{Load}| = 3|I_{out}||V_{out}| = \sqrt{3}|I_{in}||V_{out}|$$

$$\sqrt{222}\ 10 = \sqrt{3}\ \frac{\sqrt{2}}{\sqrt{3}}\ |V_{out}| \qquad |V_{out}| = \sqrt{111}\ 10 \ (rms)$$

$$|I_{out}| = \frac{|I_{in}|}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}\ \frac{1}{\sqrt{3}} = \frac{\sqrt{2}}{3} \text{ Amper} \ (rms)$$

$$S_K = 90 - 110j \text{ VA}$$

$$S_K = 3 V_{out} I_{out}^{*} = 3(X_{Load} I_{out}) I_{out}^{*} = 3|I_{out}|^2 X_{Load}$$

$$90 - 110j = 3\ \frac{2}{3}\ X_{Load}, \qquad \frac{90-110j}{2} = X_{Load} = 45 - 55j\ \Omega$$