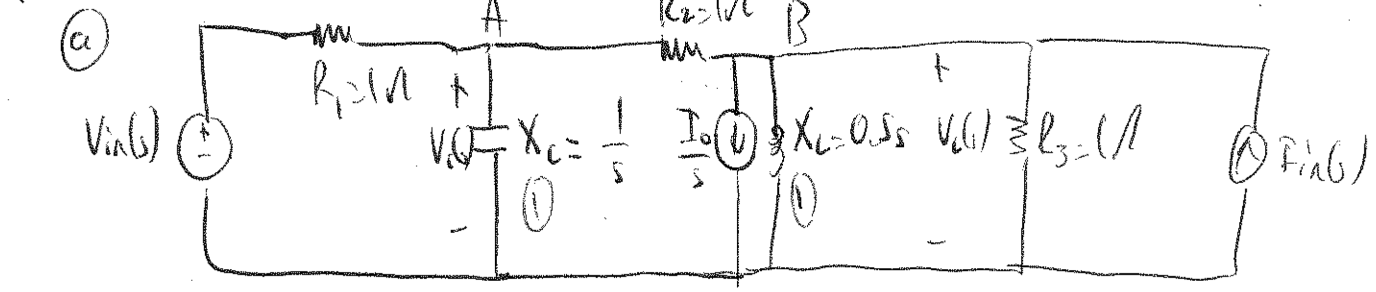
Q-1 - in Laplace domain

Devre (1)

(a)

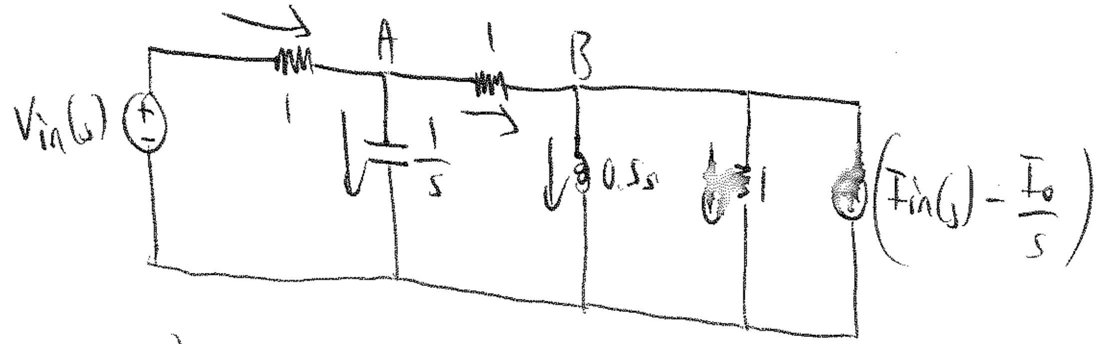


$$L \frac{dI_L}{dt} = V_L \qquad L\left[sI_L(s) - I_0\right] = V_L(s) \qquad \frac{V_L(s)}{L} + I_0 = sI_L(s)$$

(1) $I_L(s) = \dfrac{V_L(s)}{sL} + \dfrac{I_0}{s}$

(b)

Carry the current source $\frac{I_0}{s}$ near to $I_{in}(s)$



$A = V_c(s)$

5

5

$$\frac{V_{in} - A}{1} = \frac{A}{\frac{1}{s}} + \frac{A - B}{1} \qquad \frac{A - B}{1} + I_{in}(s) - \frac{I_0}{s} = \frac{B}{0.5s} + \frac{B}{1}$$

$$V_{in} - A = sA + A - B$$

$$\boxed{V_{in} = (2+s)A - B}$$

$$I_{in}(s) - \frac{I_0}{s} = \left(2 + \frac{2}{s}\right)B - A$$

$$I_{in}(s) - \frac{I_0}{s} = \frac{2s+2}{s} B - A$$

(2) $\boxed{\dfrac{s}{2s+2} I_{in}(s) - \dfrac{I_0}{2s+2} + \dfrac{sA}{2s+2} = B}$

Q-1 - (Continue)

$$V_{in} = (2+s)A - \left[\frac{s}{2s+2} I_{in} - \frac{I_0}{2s+2} + \frac{sA}{2s+2}\right]$$

$$V_{in} + \frac{s}{2(s+1)} I_{in} - \frac{I_0}{2(s+1)} = (2+s)A - \frac{sA}{2(s+1)}$$

$$V_{in} + \frac{s}{2(s+1)} I_{in} - \frac{I_0}{2(s+1)} = \frac{2(s^2+3s+2)-s}{2(s+1)} A$$

$$V_{in} + \frac{s}{2(s+1)} I_{in} - \frac{I_0}{2(s+1)} = \frac{2s^2+5s+4}{2(s+1)} A$$

(2) $$A = \frac{2(s+1)}{2s^2+5s+4} V_{in} + \frac{2s(s+1)}{2s(s+1)(2s^2+5s+4)} I_{in} - \frac{I_0}{2s^2+5s+4}$$

$$V_c(s) = \underbrace{\frac{2(s+1)}{2s^2+5s+4} V_{in} + \frac{1.s}{2s^2+5s+4} I_{in}}_{V_{c\,\text{zero-state}}} - \underbrace{\frac{I_0}{2s^2+5s+4}}_{V_{c\,\text{zero-input}}}$$

1 1

Q-2

(a) $H(j\omega) = \dfrac{10000\,(j\omega+5)}{(j\omega+100)(j\omega+500)}$ (2)

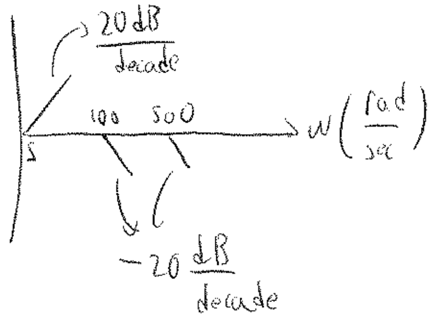
(b) $|H(j\omega)| = \dfrac{10000\,\sqrt{\omega^2+5^2}}{\sqrt{\omega^2+100^2}\,\sqrt{\omega^2+500^2}}$ (2)

(c) $|H(j\omega)|_{dB} = 20\log \dfrac{10000\,\sqrt{\omega^2+5^2}}{\sqrt{\omega^2+100^2}\,\sqrt{\omega^2+500^2}}$ (2)

(d) $\angle H(j\omega) = \mathrm{Tan}^{-1}\dfrac{\omega}{5} - \mathrm{Tan}^{-1}\dfrac{\omega}{100} - \mathrm{Tan}^{-1}\dfrac{\omega}{500}$ (3)

(e) $|H(j\omega)|\Big|_{\omega=0} = \dfrac{10000 \cdot 5}{100 \cdot 500} = 1 \qquad |H(j\omega)|_{dB}\Big|_{\omega=0} = 20\log 1 = 0$

asymptotes



(1.5) * $0<\omega<5 \Rightarrow |H(j\omega)|_{dB} \cong 0$

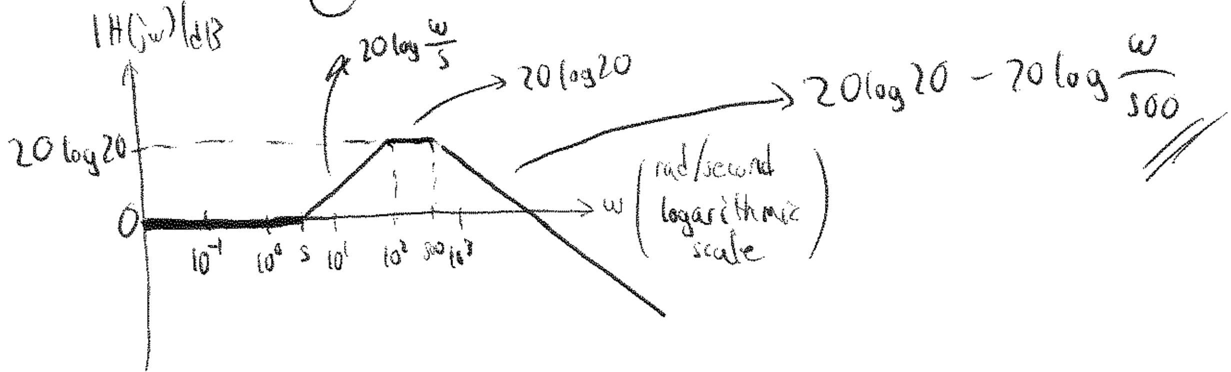
(1.5) * $5<\omega<100 \Rightarrow |H(j\omega)|_{dB} = 0 + 20\log\dfrac{\omega}{5}$

$\omega = 100 \Rightarrow |H(j\omega)|_{dB} = 20\log 20$

(1.5) * $100<\omega<500 \Rightarrow |H(j\omega)|_{dB} = 20\log 20$

(1.5) * $500<\omega \Rightarrow |H(j\omega)|_{dB} = 20\log 20 - 20\log\dfrac{\omega}{500}$

(3)



Q-2 (continue)

(f) $\angle H(j\omega)\Big|_{w=0} = 0$

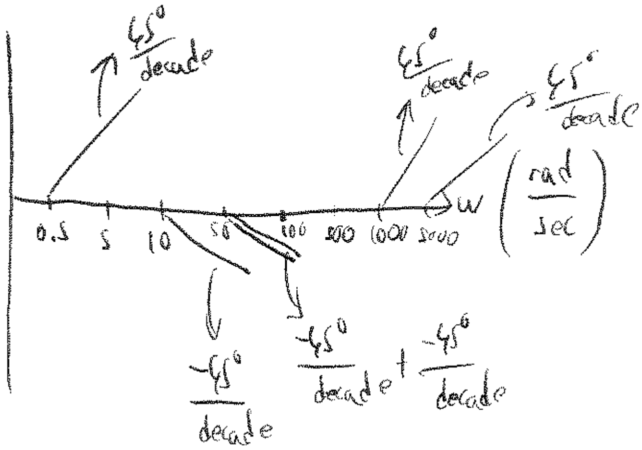
asymptotes

$s=-5$ (zero) $w = 5\,\frac{rad}{sec}$ → 0.5, 5, 50

$s=-100$ (pole) $w = 100\,\frac{rad}{sec}$ → 10, 100, 1000

$s=-500$ (pole) $w = 500\,\frac{rad}{sec}$ → 50, 500, 5000

3



1 $0 < w < 0.5 \Rightarrow \angle H(j\omega) \cong 0$

1 $0.5 < w < 10 \Rightarrow \angle H(j\omega) \cong 0 + 45\log\frac{w}{0.5}$ if $w=10$ $\angle H(j\omega) \cong 45\log 20$

1 $10 < w < 50 \Rightarrow \angle H(j\omega) \cong 45\log 20$ (constant)

1 $50 < w < 1000 \Rightarrow \angle H(j\omega) \cong 45\log 20 - 90\log\frac{w}{50}$

if $w=1000$ $\angle H(j\omega) \cong 45\log 20 - 90\log\frac{1000}{50} = 45\log 20 - 90\log 20$

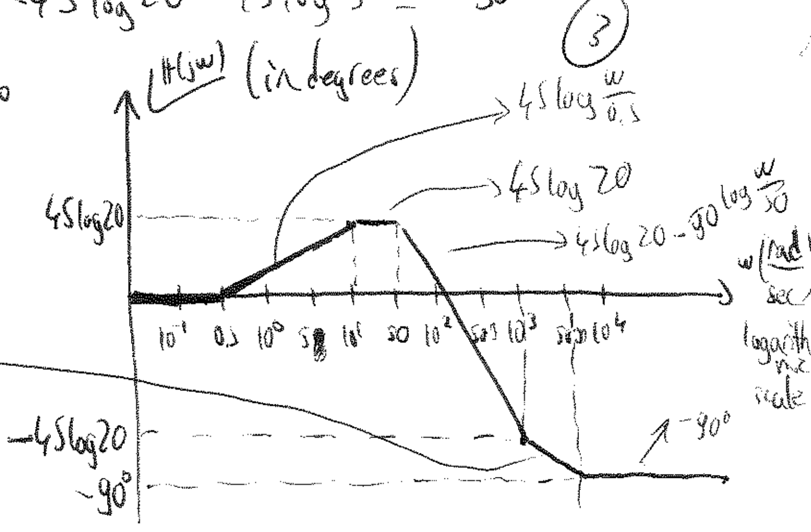
$= -45\log 20$

1 $1000 < w < 5000 \Rightarrow \angle H(j\omega) \cong -45\log 20 - 45\log\frac{w}{1000}$

if $w = 5000$ $\angle H(j\omega) \cong -45\log 20 - 45\log 5 = -90°$

1 if $w > 5000 \Rightarrow \angle H(j\omega) \cong -90°$

3



Q-3 - $H(s) = \dfrac{10100s}{(s+100)(s+10000)}$ $H(j\omega) = \dfrac{10100j\omega}{(j\omega+100)(j\omega+10000)}$

$|H(j\omega)| = \dfrac{10100\omega}{\sqrt{\omega^2+100^2}\sqrt{\omega^2+(10000)^2}}$ or $|H(j\omega)| = \dfrac{10100\omega}{\sqrt{(10^6-\omega^2)^2+(10100\omega)^2}}$

(a)

$|H(j\omega)|$ is maximized whe $\omega^2 = 10^6$ $\omega = 1000 \frac{rad}{second} = \omega_0$

resonant frequency

4

(b)

$|H(j\omega)|_{max} = \left| H(j\omega) \right| \Big|_{\omega = \omega_0 = 1000 \frac{rad}{second}} = \dfrac{10100 \times 1000}{10100 \times 1000} = 1$ //

at the half power frequencies

2

$|H(j\omega)| \Big|_{\omega=\omega_c} = \dfrac{|H(j\omega)|_{max}}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} = \dfrac{10100\omega_c}{\sqrt{(\omega_c^2+100^2)(\omega_c^2+10000^2)}} = \dfrac{1}{\sqrt{2}}$

$2 \times 10100^2\omega_c^2 = (\omega_c^2+100^2)(\omega_c^2+10000^2)$ $\omega_c^2 = x$

$2 \times 10100^2 x = (x+100^2)(x+10000^2)$ $2 \times (10100^2)x = (x+100^2)(x+10000^2)$

$2 \times (10100)^2 x = x^2 + [10000^2+100^2]x + 10^{12}$

$= (x-100^2)(x-10000^2)$ 2

$x = 100^2$ $\omega_{c_1} = 100 \frac{rad}{sec}$ 0.5

$x = 10000^2$ $\omega_{c_2} = 10000 \frac{rad}{sec}$ 0.5

Q-4-

8

$$Y(s) = \frac{5}{(s^2+1)(s^2+2s+2)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+2s+2}$$

$$5 = (As+B)(s^2+2s+2) + (Cs+D)(s^2+1)$$

$$5 = As^3 + (2A+B)s^2 + (2A+2B)s + 2B + Cs^3 + Ds^2 + Cs + D$$

$$5 = [A+C]s^3 + (2A+B+D)s^2 + (2A+2B+C)s + 2B+D$$

$A+C=0$ ① $2A+B+D=0$ ② $2A+2B+C=0$ ③ $2B+D=5$ ④

$A=-C$

$-2C+\frac{C}{2}+D=0$

$-\frac{3C}{2}+D=0$

$D=\frac{3C}{2}$

$-2C+2B+C=0$

$B=\frac{C}{2}$

$C+\frac{3C}{2}=5$

$\frac{5C}{2}=5$

$C=2$

$B=1$ 2

$D=3$

$A=-2$

1 1

$$Y(s) = \frac{-2s+1}{s^2+1} + \frac{2s+3}{s^2+2s+2} = -2\frac{s}{s^2+1} + 1\frac{1}{s^2+1} + 2\frac{(s+1)}{(s+1)^2+1^2} + 1\frac{1}{(s+1)^2+1^2}$$

$\downarrow \mathcal{L}^{-1}$

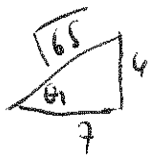
1

$$y(t) = \left[-2\cos(t) + \sin(t) + 2e^{-t}\cos(t) + e^{-t}\sin(t)\right]u(t)$$

Q-5- The sources have the same input angular frequencies ($\omega = 2 \frac{rad}{second}$)

In Phasor domain

$$V_{in}(t) = \frac{\sqrt{65}}{8} \cos(2t - \theta_1) \rightarrow \bar{V}_{in} = \frac{\sqrt{65}}{8} e^{-j\theta_1} = \frac{\sqrt{65}}{8}\left[\cos\theta_1 - j\sin\theta_1\right]$$



$$= \frac{\sqrt{65}}{8}\left[\frac{7}{\sqrt{65}} - j\frac{4}{\sqrt{65}}\right]$$

(2) $= \frac{7}{8} - \frac{4j}{8}$ Volt

$$I_{in}(t) = \frac{\sqrt{5}}{2}\cos(2t + \theta_2) \rightarrow \bar{I}_{in} = \frac{\sqrt{5}}{2} e^{j\theta_2} = \frac{\sqrt{5}}{2}\left[\frac{2}{\sqrt{5}} + j\frac{1}{\sqrt{5}}\right]$$

$\cos\theta_2 = \frac{2}{\sqrt{5}}$ $\sin\theta_2 = \frac{1}{\sqrt{5}}$ (2) $= 1 + \frac{j}{2}$ Amper

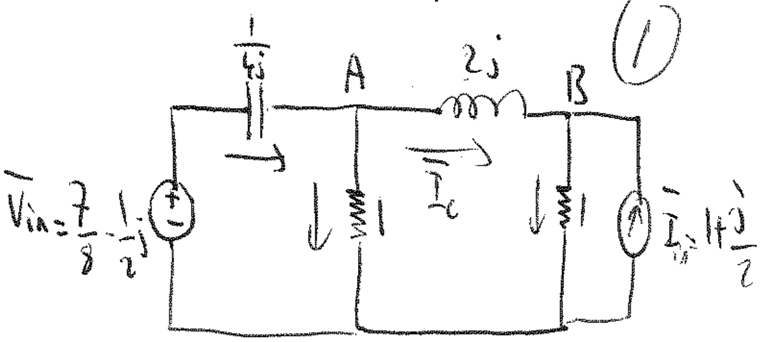
The circuit in phasor domain

$X_{R_1} = X_{R_2} = R_1 = R_2 = 1\,\Omega$ (1)

$X_C = \frac{1}{j\omega C} = \frac{1}{j \times 2 \times 2} = \frac{1}{4j}\,\Omega$ (1)

$X_L = j\omega L = j \times 2 \times 1 = 2j\,\Omega$ (1)

Circuit in phasor domain (1)



$$\frac{\bar{V}_{in} - A}{\frac{1}{4j}} = \frac{A}{1} + \frac{A - B}{2j} \quad (2)$$

$$\frac{A - B}{2j} + \underbrace{1 + \frac{j}{2}}_{\bar{I}_{in}} = \frac{B}{1} \quad (2)$$

$A = 2$ ✓ (2)

$B = 1$ ✓

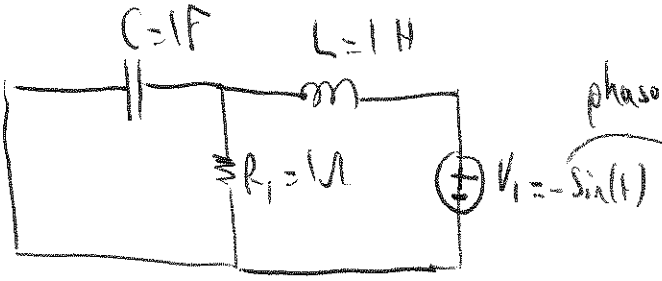
(a) $\bar{I}_C = \frac{2-1}{2j} = \frac{1}{2j} = \frac{1}{2} \cdot \frac{1}{j} = \frac{1}{2}(-j) = \frac{1}{2} e^{-j90°}$ (1)

(1) $I_C(t) = \frac{1}{2}\cos(2t - 90°) = \frac{1}{2}\sin(2t)$ Amper

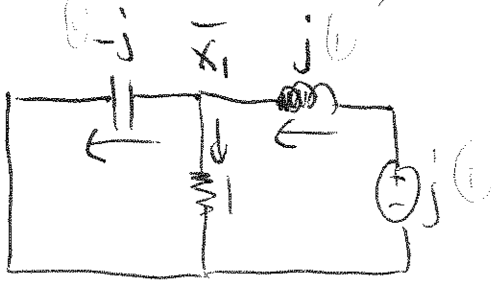
(b) $P_{R_1} = \frac{1}{2} I_{R_1} \bar{I}_{R_1} R_1 = \frac{1}{2}\left(\frac{2-0}{R_1}\right) \cdot \left(\frac{2-0}{R_1}\right)^* R_1 = \frac{1}{2} \times \frac{4}{R_1} = 2\,Watt$ (2) (2) → scenap

Q-6 — Frequencies are different $w_1 = 1 \frac{rad}{sec}$, $w_2 = 2 \frac{rad}{second}$

Kill V_2 (short V_2) (input frequency for $V_1(t)$ is $w_1 = 1 \frac{rad}{second}$)



phasor



$V_1 = -\sin(t) \rightarrow V_1 = \cos(t+90) \xrightarrow{phasor} \bar{V}_1 = e^{j90} = j$ Volt

$X_{R_1} = 1\Omega$

$X_L = j w_1 L = j \cdot 1 \cdot 1 = j$

$X_C = \frac{1}{j w_1 \cdot C} = \frac{1}{j \cdot 1 \cdot 1} = -j$

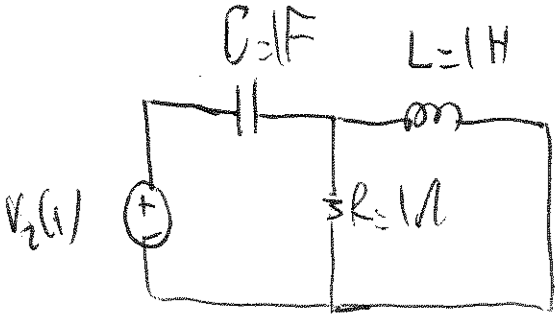
$$\frac{j - \bar{X}_1}{j} = \frac{\bar{X}_1}{1} + \frac{\bar{X}_1}{-j}$$

$$1 = \frac{\bar{X}_1}{j} + \bar{X}_1 - \frac{\bar{X}_1}{j}$$

$$1 = \bar{X}_1$$

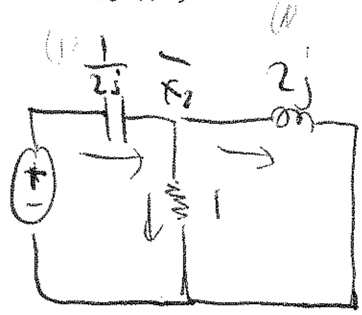
$$X_1(t) = \cos(t)$$

Kill V_1 (short V_1) (input frequency for $V_2(t)$ is $w_2 = 2 \frac{rad}{second}$)

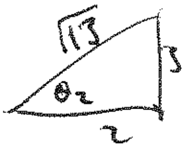


phasor

$\bar{V}_2 = \frac{1}{2} + j\frac{3}{4}$



$V_2 = \frac{\sqrt{13}}{4}\cos(2t + \theta_2) \xrightarrow{phasor} \frac{\sqrt{13}}{4}(\cos\theta_2 + j\sin\theta_2)$



$= \frac{\sqrt{13}}{4}\left[\frac{2}{\sqrt{13}} + j\frac{3}{\sqrt{13}}\right]$

$\bar{V}_2 = \frac{1}{2} + j\frac{3}{4}$ Volt

$X_{R_1} = 1$

$X_L = j w_2 L = 2j$

$X_C = \frac{1}{j w_2 C} = \frac{1}{2j}$

$$\frac{\bar{V}_2 - \bar{X}_2}{\frac{1}{2j}} = \frac{\bar{X}_2}{1} + \frac{\bar{X}_2}{2j}$$

$$\frac{\frac{1}{2} + j\frac{3}{4} - \bar{X}_2}{\frac{1}{2j}} = \frac{\bar{X}_2}{1} + \frac{\bar{X}_2}{2j}$$

Q-6- continue

$$j+\left(\frac{j^2 3}{2}\right)-2j\bar{X}_2=\bar{X}_2+\frac{\bar{X}_2}{2j}$$

$$j-\frac{3}{2}=\bar{X}_2\left[1+\frac{1}{2j}+2j\right] \longrightarrow j-\frac{3}{2}=\bar{X}_2\left[1+\frac{3}{2}j\right]$$

(1) $\bar{X}_2=j \longrightarrow X_2(t)=\cos(2t+90)$ (0.5)

$=e^{j90}$

(a) At SSS (Sinusoidal Steady State)

$$A=X_1(t)+X_2(t)=\cos(t)+\cos(2t+90)$$
$$=\cos(t)-\sin(2t)\text{ Volt}$$ (1)

$$I_{R_1}=\frac{A}{R_1}=\frac{\cos(t)-\sin(2t)}{1}=\cos(t)-\sin(2t)\text{ Amper}$$

(b) $$P_{R_1,SSS}=\frac{1}{2}\,\frac{\bar{X}_1\bar{X}_1^*}{R_1}+\frac{1}{2}\,\frac{\bar{X}_2\bar{X}_2^*}{R_1}$$ (1) (1)

$$=\frac{1}{2}\,\frac{1\times 1}{1}+\frac{1}{2}\,\frac{(j)\cdot(-j)}{1}=\frac{1}{2}+\frac{1}{2}=1\text{ Watt}$$ (2)