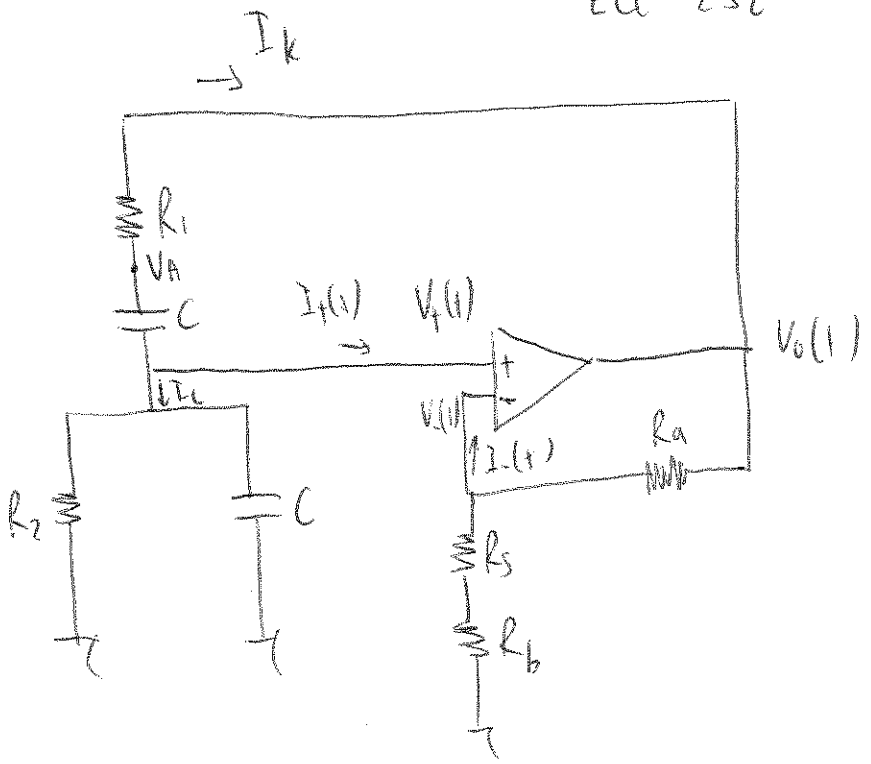
Sinusoidal Oscillator
ECE 232



$$I_+(1) = I_-(1) \approx 0 \qquad V_+(1) \approx V_-(1)$$

$$V_-(1) = V_+(1) = V_o \frac{R_s + R_b}{R_a + R_s + R_b}$$

$$I_K + I_L + I_+ = 0 \qquad I_+ \approx 0 \qquad I_K + I_L = 0$$

$$I_L = \frac{V_+}{R_2} + C\frac{dV_+}{dt} \qquad I_K = C\frac{d(V_+ - V_A)}{dt} = \frac{V_A - V_o}{R_1}$$

$$\mathcal{L}\left(I_L = V_+\left[\frac{1}{R_2} + sC\right] \right) \qquad I_K = C\left[sV_+ - sV_A\right] = \frac{V_A - V_o}{R_1}$$

$$\Rightarrow \quad CR_1 sV_+ + V_o = (CR_1 s + 1)V_A$$

$$V_A = \frac{CR_1 sV_+ + V_o}{CR_1 s + 1}$$

$$I_L = -I_K$$

$$V_+\left[\frac{1+sR_2C}{R_2}\right] = \frac{V_0 - V_A}{R_1} = \frac{V_0 - \frac{CR_1sV_+ + V_0}{CR_1s+1}}{R_1}$$

$$V_+\left[\frac{1+sR_2C}{R_2}\right] = \frac{V_0CR_1s - V_+CR_1s}{R_1[CR_1s+1]}$$

$$R_1V_+\left[1+sR_2C\right]\left[1+sR_1C\right] = R_2\left[V_0CR_1s - V_+CR_1s\right]$$

$$V_+\left[1+s[R_2+R_1]C+R_1R_2C^2\right] = \left(V_0Cs - V_+Cs\right)R_2 \quad \text{pero } V_+ = \frac{R_3+R_b}{R_a+R_3+R_b}V_0$$

$$V_0\frac{R_3+R_b}{R_a+R_3+R_b}\left[1+s[R_2+R_1]C+R_1R_2C^2\right] = Cs\left[V_0 - \frac{R_3+R_b}{R_a+R_3+R_b}V_0\right]R_2$$

$$V_0\frac{R_3+R_b}{R_a+R_3+R_b}\left[1+s[R_2+R_1]C+R_1R_2C^2\right] = V_0Cs\left[\frac{R_a}{R_a+R_3+R_b}\right]R_2$$

$$V_0\left[R_3+R_b\right]\left[1+s(R_2+R_1)C+R_1R_2C^2\right] = V_0CsR_aR_2$$

$$V_0\left[1+s(R_2+R_1)C+R_1R_2C^2\right] = V_0Cs\frac{R_aR_2}{R_3+R_b}$$

$$V_0\left[1+s\left(R_2+R_1-\frac{R_aR_2}{R_3+R_b}\right)C+R_1R_2C^2\right] = 0$$

$$V_0\left(1+s\left[R_2+R_1-\frac{R_a R_2}{R_3+R_b}\right]+R_1R_2C^2s^2\right)=0$$

if $R_2+R_1=\frac{R_aR_2}{R_3+R_b}$ the equation turns to

$$V_0\left[1+R_1R_2C^2s^2\right]=0$$

in time domain this equation gives us

$$V_0+R_1R_2C^2\frac{d^2V_0}{dt^2}=0 \quad \text{or} \quad \frac{d^2V_0}{dt^2}+\frac{1}{R_1R_2C^2}V_0=0 \quad (*)$$

Hence the solution of * is

$$V_0(t)=k_1\sin(\omega t)+k_2\cos(\omega t)$$

where $\omega=\frac{1}{C\sqrt{R_1R_2}}$ (angular frequency of sinusoidal oscillator)