ECE 232 Midterm 2014 - 2015

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(Q1) In Time Domain

$$C\frac{dV_c}{dt} = I_c \xrightarrow{\mathcal{L}} C[sV_c(s) - V_c(0)] = I_c(s)$$

$$I_c(s) = 0.2[sV_c(s) - V_c(0)]$$

$$V_{in} = V_R + V_L + V_C$$

$$V_{in} = RI_L + L\frac{dI_L}{dt} + V_C$$

$\mathcal{L}\{$

$$V_{in}(s) = RI_L(s) + L[sI_L(s) - I_L(0)] + V_C(s)$$

$$V_{in}(s) = 6I_L(s) + [sI_L(s) - I_L(0)] + V_C(s)$$

diff denklemler 5

$$V_{in}(s) = [6+s]I_L(s) - I_L(0) + V_C(s)$$

$$V_{in}(s) = [6+s]\,0.2[sV_c(s) - V_c(0)] - I_L(0) + V_C(s)$$

$$V_{in}(s) = [0.2s^2 + 1.2s + 1]V_C(s) - 0.2[6+s]V_C(0) - I_L(0)$$

$$V_C(s) = \frac{0.2(6+s)V_C(0) + I_L(0)}{0.2s^2 + 1.2s + 1} + \frac{V_{in}(s)}{0.2s^2 + 1.2s + 1}$$

laplace denklemler 5

$$V_C(s) = \frac{(6+s)V_C(0) + 5I_L(0)}{s^2 + 6s + 5} + \frac{5\,V_{in}(s)}{s^2 + 6s + 5}$$

(a) If $I_L(0) = 0$ Ampere and $V_C(0) = 0$ Volt

$$V_C(s) = \frac{5}{s^2 + 6s + 5}V_{in}(s) \qquad \text{if } V_{in}(t) = u(t) \xrightarrow{\mathcal{L}} V_{in}(s) = \frac{1}{s}$$

2 puan

$$V_C(s) = \frac{5}{s^2 + 6s + 5}\cdot\frac{1}{s} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+5}$$

$(s+1)(s+5) \quad (s)(s+5) \quad s(s+1)$

6 puan

$$5 = A(s+1)(s+5) + B(s)(s+5) + C\,s(s+1)$$

$$5 = (A+B+C)s^2 + [6A+5B+C]s + 5A$$

$A = 1$

$$B = -\frac{5}{4}$$

$A+B+C=0 \longrightarrow B+C+1=0 \quad B+C=-1$

$6A+5B+C=0 \longrightarrow 6A+5B+C=0 \quad 5B+C=-6$

$$C = \frac{1}{4}$$

$A = 1 \quad B = -\frac{5}{4} \quad C = \frac{1}{4}$

$$V_c(s) = \frac{1}{s} - \frac{5}{4}\frac{1}{s+1} + \frac{1}{4}\frac{1}{s+5} \rightarrow V_c(t) = u(t) - \frac{5}{4}e^{-t}u(t) + \frac{1}{4}e^{-5t}u(t)$$

$$= \left[1 - \frac{5}{4}e^{-t} + \frac{1}{4}e^{-5t}\right]u(t)$$

2 puan

(b) If $V_{in}(t) = 0$ $\quad V_c(0) = 2\,V_0 lt \quad I_L(0) = 0$ Amper

$$V_c(s) = \frac{2(s+6)+0}{s^2+6s+5} = \frac{2s+12}{s^2+6s+5} = \frac{A}{s+5} + \frac{B}{s+1}$$

$(s+1) \quad (s+5)$

2 puan

$2s+12 = A(s+1) + B(s+5)$

$2s+12 = (A+B)s + (A+5B) \quad A+B = 2 \quad A+5B = 12 \quad B = \frac{5}{2}$

$A = -\frac{1}{2}$

$$V_c(s) = -\frac{1}{2}\frac{1}{s+5} + \frac{5}{2}\frac{1}{s+1} = \left[-\frac{1}{2}e^{-5t} + \frac{5}{2}e^{-t}\right]u(t)$$

3 puan

Q2

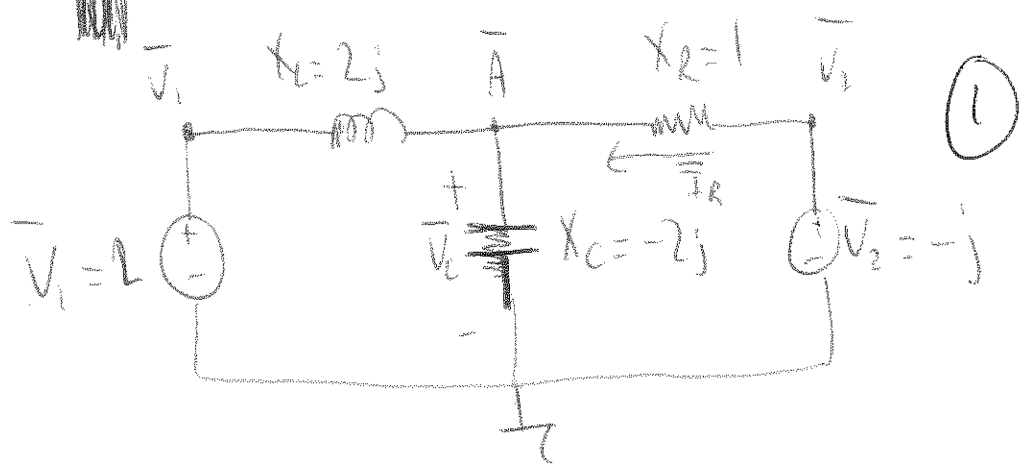
(a) $V_1(t) = 2\cos(t)$ $\downarrow$ $w_1 = 1\,\frac{rad}{sec}$ $\qquad$ $V_2(t) = \sin(t)$ $\downarrow$ $w_2 = 1\,\frac{rad}{sec}$ $\qquad$ $w = w_1 = w_2 = 1\,\frac{rad}{sec}$ (same)

Use phasors

$\bar{V}_1 = 2$, $\bar{V}_2 = -j$, $X_R = 1$, $X_L = jw_1L = jwL = 2j$

$X_C = \frac{1}{jw_2C} = \frac{1}{jwC} = \frac{1}{j\cdot 1\times 0.5} = -2j$

(3)



(1)

$$\frac{\bar{V}_1 - \bar{A}}{2j} + \frac{\bar{V}_2 - \bar{A}}{1} = \frac{\bar{A}}{X_C} \qquad \frac{2 - \bar{A}}{2j} + \frac{-j - \bar{A}}{1} = \frac{\bar{A}}{-2j}$$

(3)

$(2j)$ $\quad$ (-1)

$$2 - \bar{A} + 2 - 2j\bar{A} = -\bar{A} \qquad 4 = 2j\bar{A} \qquad \bar{A} = \frac{2}{j} = -2j$$

(1) $\bar{V}_c = \bar{A} = -2j \Rightarrow V_c(t) = -2\cos(t + 90^\circ)$ Volt (At sinusoidal steady-state)

$$\bar{I}_R = \frac{\bar{V}_2 - \bar{A}}{X_R} = \frac{-j - (-2j)}{1} = \frac{j}{1} = j \qquad P_R = \frac{1}{2} R \bar{I}_R \bar{I}_R^* = \frac{1}{2}\times 1 (j)(j^*)$$

(1)

$$= \frac{1}{2}\times 1\times j\times(-j) = \frac{1}{2}\ \text{Watt}$$

(1)

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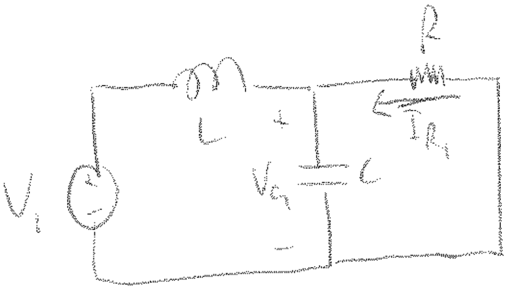
(b) $V_1(t) = \cos(t)$ $\qquad$ $V_2(t) = \sin(2t)$

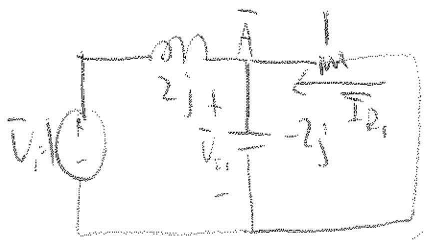
$\downarrow$ $\qquad$ $\downarrow$

$w_1 = 1\ \frac{rad}{sec}$ $\qquad$ $w_2 = 2\ \frac{rad}{sec}$ $\qquad$ $w_1 \neq w_2$

Kill $V_2(t)$ $\quad$ $\left[V_1(t) = \cos(t) \right]$ $\quad$ $w_1 = 1\ \frac{rad}{sec}$



Phasor circuit



$\xrightarrow{\text{Phasor}}$

$X_L = jw_1L = 2j\ \Omega$

$X_C = \frac{1}{jw_1C} = \frac{1}{j \times 1 \times 0.5} = -2j\ \Omega$

3puan

$X_R = 1\ \Omega$

$\bar{V}_1 = 1$

$$\frac{\bar{V}_1 - \bar{A}}{2j} = \frac{\bar{A}}{-2j} + \frac{\bar{A}}{1}$$

$(-1) \quad (2j)$

2puan

$\bar{V}_1 - \bar{A} = -\bar{A} + \bar{A}2j \rightarrow 1 = 2j\bar{A}$

$\bar{A} = \frac{1}{2j} = \frac{-j}{2} = -\frac{j}{2} = \bar{V}_{c_1}$ $\rightarrow$ 1puan

$V_{c_1}(t) = -\frac{1}{2}\cos(t + 90^\circ)$ Volt

1puan

$\bar{I}_{R_1} = \frac{0 - \bar{A}}{1} = \frac{0 - \left(-\frac{j}{2}\right)}{1} = \frac{j}{2}$

$P_{R_1} = \frac{1}{2} R \bar{I}_{R_1} \bar{I}_{R_1}^{*} = \frac{1}{2} \times 1 \times \left(\frac{j}{2}\right)\left(-\frac{j}{2}\right)$

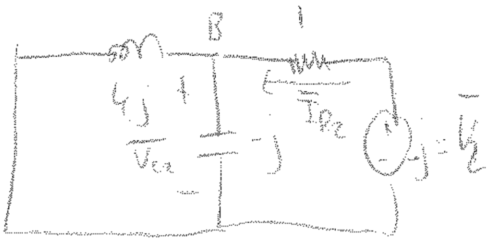
$= \frac{1}{8}$ Watt

1puan

Kill $V_1(t)$ $\quad \left[V_2(t) = \sin(2t)\right]$

$\omega_2 = 2\ \frac{rad}{sec}$

Phasor circuit



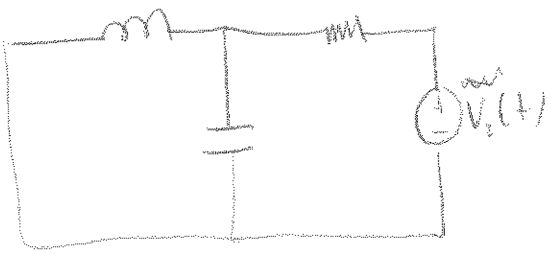
Phasor $\longrightarrow$

$\bar{V}_2 = -j$ Volt

$X_R = 1\Omega$

$X_C = \frac{1}{j\omega_2 C} = \frac{1}{j \times 2 \times 0.5} = -j$

$X_L = j\omega_2 L = 4j$



3 puan

3 puan

$$\frac{\bar{V}_2 - \bar{B}}{1} = \frac{\bar{B}}{-j} + \frac{\bar{B}}{4j}$$

$(4j) \quad (-4)$

$$(-j - \bar{B})4j = -4\bar{B} + \bar{B}$$

$$4 = [4j - 3]\bar{B}$$

$$\bar{B} = \frac{4}{4j-3} = \frac{-4}{3-4j}$$

$$\bar{B} = \frac{-4(3+4j)}{(3-4j)(3+4j)} = \frac{-4(3+4j)}{25} = -\frac{4}{5}\left[\frac{3+4j}{5}\right]$$

1 puan

$$\bar{V}_{C_2} = \bar{B} \implies V_{C_2}(t) = -\frac{4}{5}\cos\left(2t + \tan^{-1}\frac{4}{3}\right)$$

1 puan

$$\bar{I}_{R_2} = \frac{-j - \bar{B}}{1} = \frac{-j - \left[-\frac{4}{25}(3+4j)\right]}{1} = \frac{-j + \frac{4}{25}(3+4j)}{1} = \frac{-25j + 12 + 16j}{25}$$

$$\bar{I}_{R_2} = \frac{12 + 9j}{25} = \frac{3}{5}\left[\frac{4+3j}{5}\right]$$

$$P_{R_2} = \frac{1}{2} R \bar{I}_{R_2} \bar{I}_{R_2}^* = \frac{1}{2} \times 1 \times \frac{3}{5}\left[\frac{4+3j}{5}\right] \times \frac{3}{5} \times \left[\frac{4-3j}{5}\right] = \frac{3 \times 3 \times 25}{2 \times 25 \times 25} = \frac{9}{50} \text{ Watt}$$

1 puan

$V_c(t) = V_{c_1}(t) + V_{c_2}(t)$

$= -\frac{1}{2}\cos(t + 90°) - \frac{4}{5}\cos\left(2t + \tan^{-1}\frac{4}{3}\right)$ 1 puan

$P_R = P_{R_1} + P_{R_2} = \frac{1}{8} + \frac{9}{50} = \frac{25+36}{200} = \frac{61}{200}$ Watt 1 puan

Q3 $H(s) = 5\frac{(s+10)}{(s+50)}$

$H(j\omega) = 5\frac{(j\omega+10)}{j\omega+50}$

(a) $|H(j\omega)| = 5\frac{\sqrt{\omega^2+10^2}}{\sqrt{\omega^2+50^2}}$

(3)

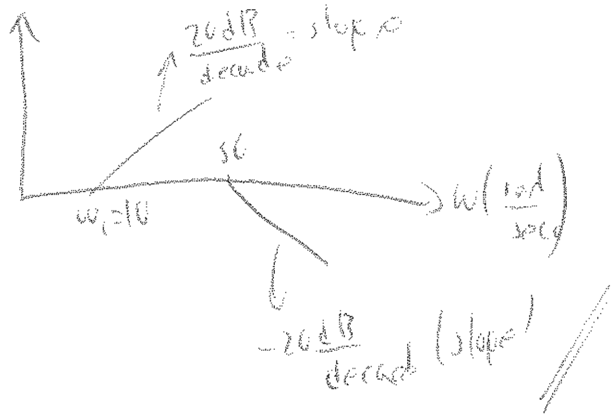
(b) $\angle H(j\omega) = \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{\omega}{50}$

(3)

(c) zero $z_1 = -10 \Rightarrow$ asymptote $\omega_1 = 10\,\frac{rad}{sec}$

pole $p_1 = -50 \Rightarrow \omega_2 = 50\,\frac{rad}{sec}$

effects of asymptotes



if $0 < \omega < 10 = \omega_1$

$|H(j\omega)| \cong 5 \times \frac{10}{50} = 1 \Rightarrow 20\log|H(j\omega)| \cong |H(j\omega)|_{dB} \cong 20\log 1 = 0\,dB$ (3)

if $\underset{10}{\omega_1} < \omega < \underset{50}{\omega_2} \Rightarrow$

$|H(j\omega)| \cong 5\frac{\omega}{50} = \frac{\omega}{10} \Rightarrow 20\log|H(j\omega)| = |H(j\omega)|_{dB} = 20\log\omega - 20\log 10$

(3)

$|H(j\omega)|_{dB,\ \omega=\omega_1} \cong 20\log 10 - 20\log 10 = 0\,dB$

$|H(j\omega)|_{\omega=\omega_2} \cong 20\log 50 - 20\log 10 = 20\log 5$

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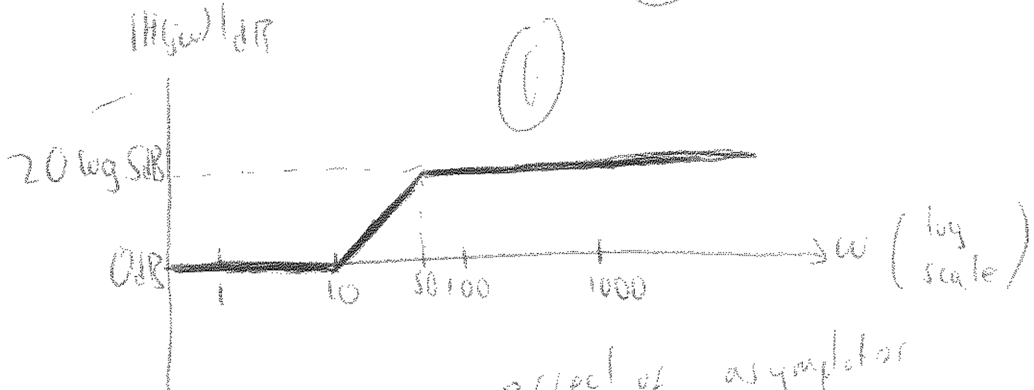
if $\omega < \omega_2 = 50$

$|H(j\omega)| \cong 5\frac{\omega}{\omega} = 5 \quad |H(j\omega)|_{dB} = 20\log|H(j\omega)| \cong 20\log 5$

(2)

(1)

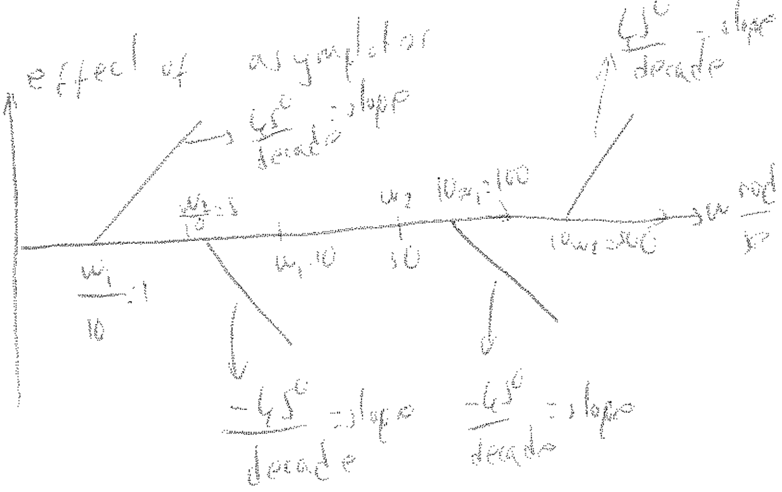


(d)

zero $z=-10 \Longrightarrow \omega_1 = 10 \frac{rad}{sec}$ asymptote

pole $p=-50 \Longrightarrow \omega_2 = 50 \frac{rad}{sec}$

(3)



$0 < \omega < \frac{\omega_1}{10} = 1 \quad \angle H(j\omega) = 0$ (1)

$1 = \frac{\omega_1}{10} < \omega < \frac{\omega_2}{10} = 5 \quad \angle H(j\omega) \cong 0 + 45\log\frac{\omega}{\frac{\omega_1}{10}} = 45\log\frac{\omega}{\frac{10}{10}} = \cancel{}$

$= 45\log\omega$

$\omega = \frac{\omega_1}{10} = 1 \quad \angle H(j\omega) = 0^0$ (2)

$\omega = \frac{\omega_2}{10} = 5 \quad \angle H(j\omega) = 45\log 5$

$\frac{\omega_2}{10} < \omega < 10\omega_1 \quad \angle H(j\omega) = 45\log 5$ (1)

$5 < \omega < 100$

$10\omega_1 < \omega < 10\omega_2$ $\quad \angle H(j\omega) = 45\log 5 - 45\log\frac{\omega}{10\omega_1} = 45\log 5 - 45\log\frac{\omega}{100}$

$100 < \omega < 500$

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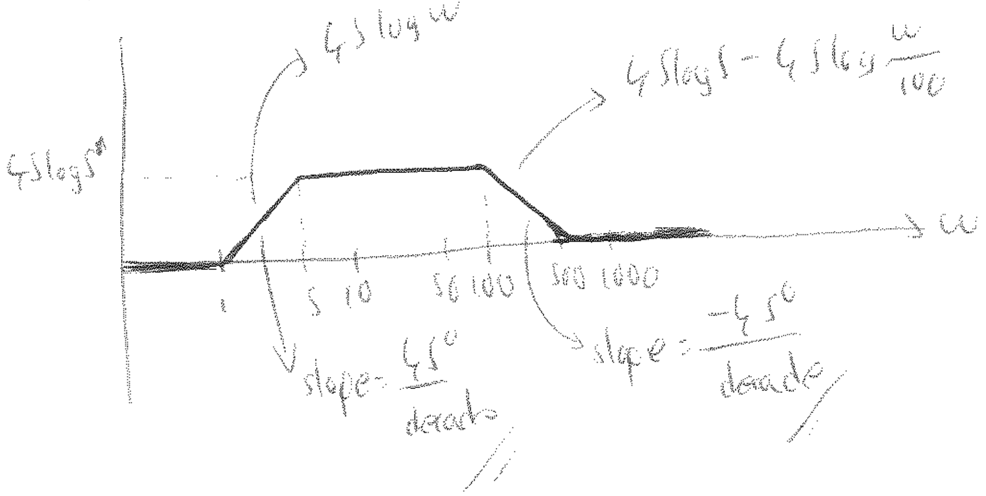
2

$\omega = 500 \quad \angle H(j\omega) = 45\log 5 - 45\log\frac{500}{100} = 0^\circ$

$500 = 10\omega_2 < \omega \Rightarrow \angle H(j\omega) = 0^\circ$

$\angle H(j\omega)$ (degree)

=



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(4) $$H(s) = \frac{s^2+1600}{s^2+100s+1600} = \frac{s^2+1600}{s^2+2D\omega_0 s+\frac{40^2}{\omega_0^2}}$$

(a) $$H(j\omega) = \frac{1600-\omega^2}{1600-\omega^2+100j\omega}$$

$$|H(j\omega)| = \frac{|1600-\omega^2|}{\sqrt{[1600-\omega^2]^2+[100\omega]^2}}$$ (3)

(b) $0<\omega<40 \Rightarrow \angle H(j\omega) = -\tan^{-1}\frac{100\omega}{1600-\omega^2}$ (3)

$40<\omega \Rightarrow \angle H(j\omega) = 180 - \tan^{-1}\frac{100\omega}{1600-\omega^2}$

(c) If $|H(j\omega)|$ is minimized $|1600-\omega^2| = 0$ $\omega = 40\ \frac{rad}{sec}$

if $\omega = 40\ \frac{rad}{sec}$ $|H(j\omega)| = 0$ (2)

(d) $2D\underbrace{\omega_0}_{40} = 100$ $2D40 = 100$ $D = \frac{100}{80} = \frac{5}{4} = 1.25$ (2)

(e) if $\omega = 0$ $|H(j\omega)| = 1 = |H(j\omega)|_{max}$

if $\omega \to \infty$ $|H(j\omega)| = 1 = |H(j\omega)|_{max}$

at $\omega_{c_1}, \omega_{c_2}$ $$|H(j\omega)|\Big|_{\omega=\omega_c} = \frac{|H(j\omega)|_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\frac{|1600-\omega_c^2|}{\sqrt{(1600-\omega_c^2)^2+(100\omega_c)^2}} = \frac{1}{\sqrt{2}}$$ (2)

$$2(1600-w_{c_{mid}}^2)^2 = (1600-w_{c_{1,2}}^2)^2 + (100w_{c_{1,2}})^2$$

$$(1600-w_c^2)^2 = (100w_c)^2 \qquad w_c^2 \mp 100w_c - 1600 = 0$$

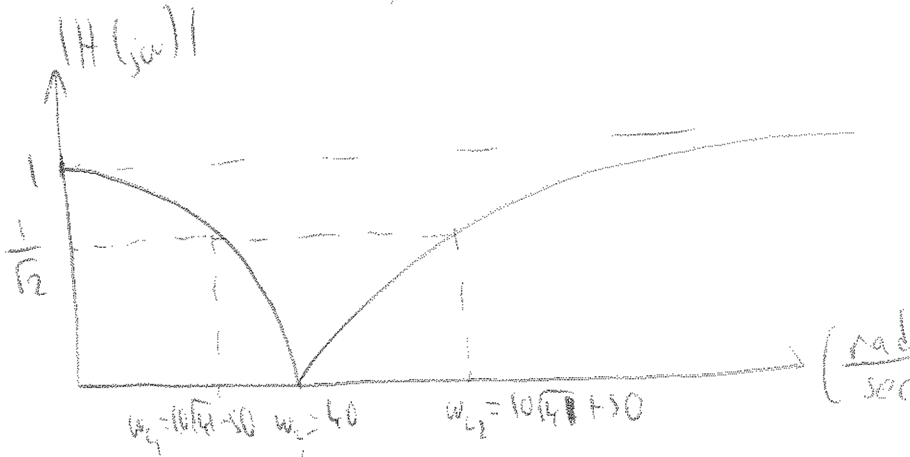
$$1600 - w_c^2 = \mp 100 w_c$$

$$w_c = \frac{\pm 100 \mp \sqrt{(100)^2 + 4 \cdot 1600}}{2} = \frac{\pm 100 \mp \sqrt{16400}}{2}$$

$$w_{c_2} = 50 + 5\sqrt{164} \qquad w_{c_1} = 5\sqrt{164} - 50$$

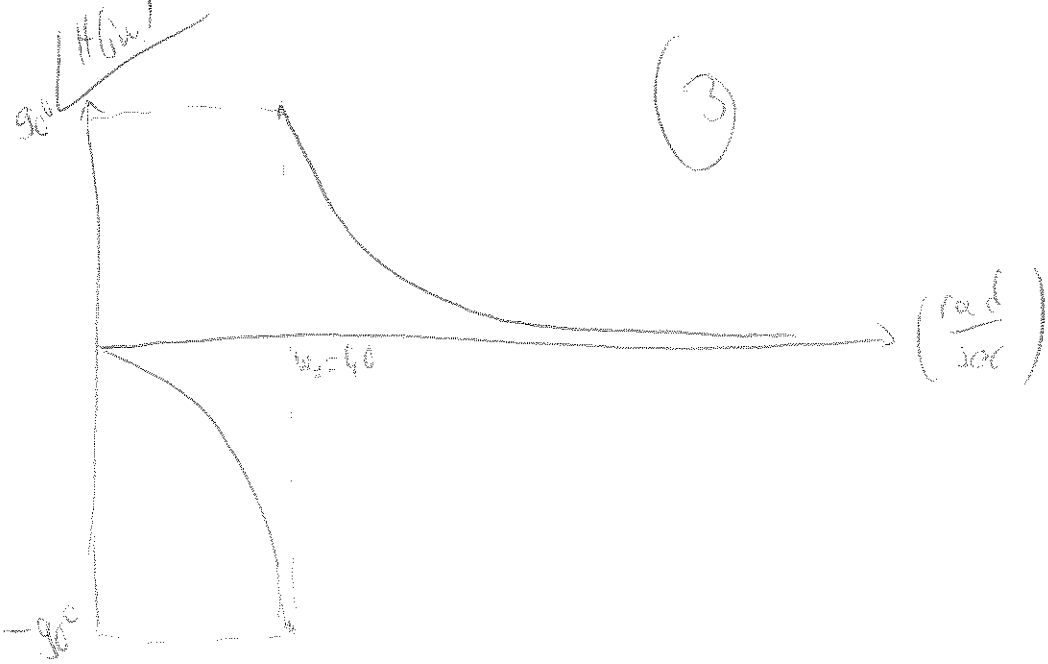
$$w_{c_2} = 50 + 10\sqrt{41} \quad (2) \quad w_{c_1} = 10\sqrt{41} - 50$$

f)



(3)

g)



(3)