ECE 232 Final [2012-2013] solutions

Q1

$V_1 = nV_2$

$I_2 = -nI_1$

$X_C = \frac{1}{s}$

$X_L = s$

$n = 0.2$

$V_2 = \left[\frac{1}{s} + s\right](-I_2)$

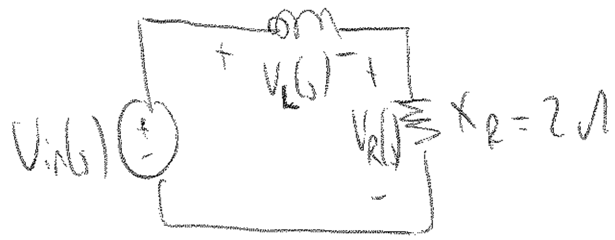
$V_2 = \left[\frac{1+s^2}{s}\right] nI_1$

$\frac{V_1}{n} = \frac{1+s^2}{s} nI_1 \Rightarrow \frac{V_1}{I_1} = \left[\frac{1+s^2}{s}\right] n^2$

$\frac{V_1}{I_1} = 0.04\left[\frac{1+s^2}{s}\right]$ //

Q2

In Laplace domain



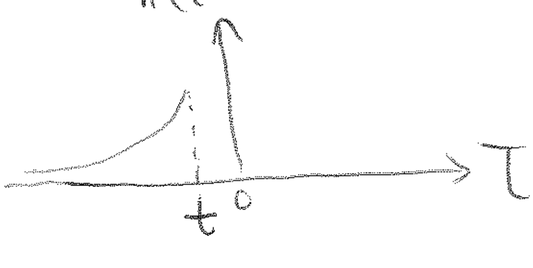
(a) $H(s) = \frac{V_R(s)}{V_{in}(s)} = \frac{2}{2s+2} = \frac{1}{s+1}$

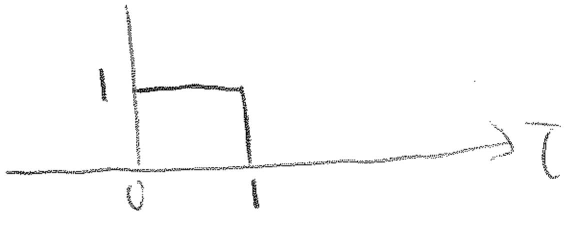
(b) $H(s) = \frac{1}{s+1} \Rightarrow h(t) = \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}u(t)$

(c) $V_R(t) = \int_{-\infty}^{t} V_{in}(t-\tau)\, h(\tau)\, d\tau = \int_{-\infty}^{t} h(t-\tau)\, V_{in}(\tau)\, d\tau$

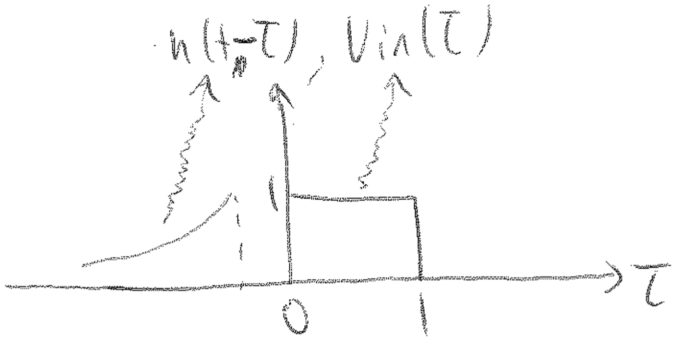
$h(t-\tau) = e^{-(t-\tau)} u(t-\tau)$

$V_{in}(\tau) = u(\tau) - u(\tau - 1)$



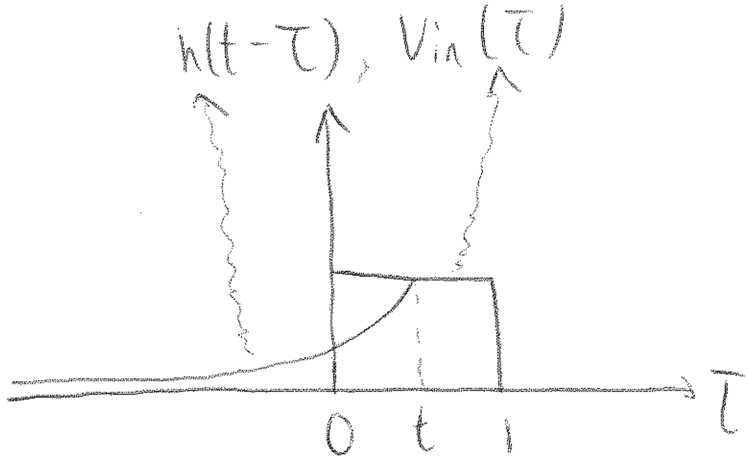


CASE I if $t<0$



$$V_R(t)=\int_{-\infty}^{t} h(t-\tau)\,V_{in}(\tau)\,d\tau = 0$$

Case II $0\leq t<1$



$$V_R(t)=\int_{-\infty}^{t} h(t-\tau)\,V_{in}(\tau)\,d\tau$$

$$=\int_{0}^{t} e^{-(t-\tau)}\,u(t-1)\,1\,d\tau$$

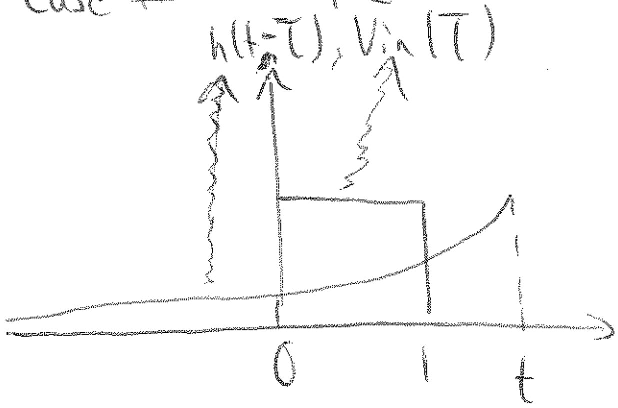
$$=\int_{0}^{t} e^{-t}e^{\tau}\,d\tau$$

$$=e^{-t}\int_{0}^{t} e^{\tau}\,d\tau$$

$$=e^{-t}\left[e^{\tau}\Big|_0^t\right]=e^{-t}\left[\left[e^{t}-1\right]\right]$$

$$=1-e^{-t}$$

Case III $1\leq t$



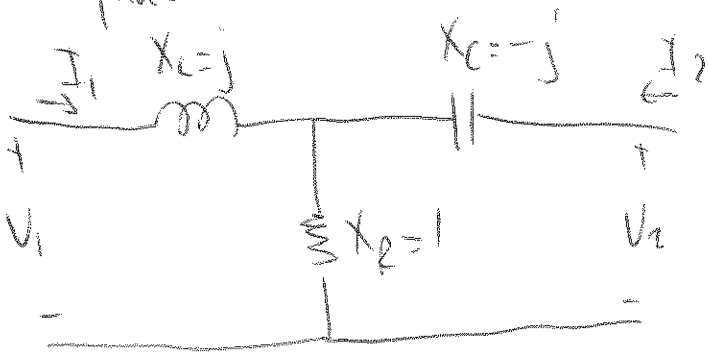
$$V_R(t)=\int_{-\infty}^{t} h(t-\tau)\,V_{in}(\tau)\,d\tau$$

$$=\int_{0}^{1} e^{-(t-\tau)}\,d\tau$$

$$=e^{-t}\int_{0}^{1} e^{\tau}\,d\tau=e^{-t}\left[e^{\tau}\Big|_0^1\right]$$

$$V_R(t)=e^{-t}\left[e-1\right]$$

Q3 In phasor domain



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$V_1 = X_L I_1 + X_R (I_1 + I_2)$$

$$V_2 = X_C I_2 + X_R (I_1 + I_2)$$

$$V_1 = j I_1 + 1(I_1 + I_2)$$

$$V_2 = (-j) I_2 + 1(I_1 + I_2)$$

$$V_1 = (j+1) I_1 + I_2$$

$$V_2 = I_1 + (1-j) I_2$$

$\Rightarrow$ $z_{11} = j+1$, $z_{12} = 1$, $z_{21} = 1$, $z_{22} = 1-j$

(Q4) $|S_T| = \sqrt{P_T^2 + Q_T^2} = 100\sqrt{2}$ VA

$|S_T| = 3\,|V_{in}|\,|I_{in}|$

$100\sqrt{2} = 3 \times 100 \times |I_{in}| \Rightarrow |I_{in}| = \frac{\sqrt{2}}{3}$ Ampere (rms)

$P_{loss\_line} = 3\,|I_{in}|^2\,Re\{X_{line}\} = 3\,\frac{2}{9}\,5 = \frac{10}{3}$ Watt

$Q_{loss\_line} = 3\,|I_{in}|^2\,Im\{X_{line}\} = 3\,\frac{2}{9}\,5 = \frac{10}{3}$ VAR

$P_{Load} = P_T - P_{Loss\_line} = 100 - \frac{10}{3} = \frac{290}{3}$ Watt

$Q_{Load} = Q_T - Q_{Loss\_line} = 100 - \frac{10}{3} = \frac{290}{3}$ VAR

$|S_{Load}| = \sqrt{P_{Load}^2 + Q_{Load}^2} = \frac{290}{3}\sqrt{2}$ VA

$|S_{Load}| = \sqrt{3}\,|V_{out}|\,|I_{in}|$

$\frac{290\sqrt{2}}{3} = \sqrt{3}\,\frac{\sqrt{2}}{3}\,V_{out} \qquad V_{out} = \frac{290}{\sqrt{3}}$ Volt (rms)

$|I_{out}| = \frac{|I_{in}|}{\sqrt{3}} = \frac{\sqrt{2}}{3\sqrt{3}}$ Ampere (rms)

$Re\{X_{Load}\} = \frac{P_{Load}}{3\,|I_{out}|^2} = \frac{\frac{290\sqrt{2}}{3}}{3\,\frac{2}{27}}\,\Omega, \quad Im\{X_{Load}\} = \frac{Q_{Load}}{3\,|I_{out}|^2} = \frac{\frac{290\sqrt{2}}{3}}{3\,\frac{2}{27}}$

$X_{Load} = \frac{870}{\sqrt{2}} + j\,\frac{870}{\sqrt{2}}\,\Omega$