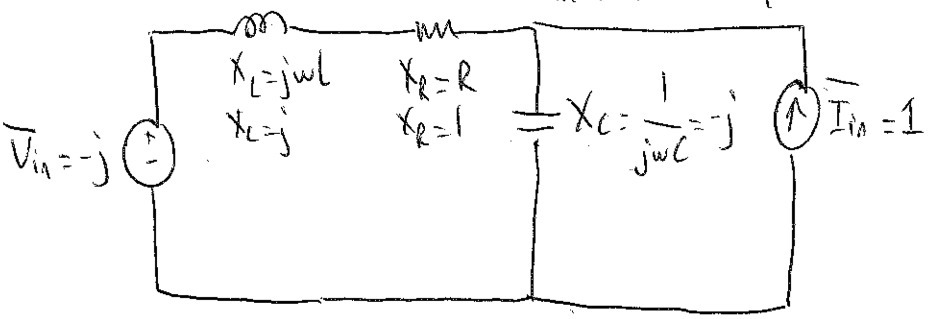
2013-2014 ECE 232 Midterm solutions

Q-1- In phasor domain $V_{in}(t) = \sin(\omega t) = \cos(\omega t - 90)$ Volt $\xrightarrow{phasor}$ $\bar{V}_{in} = e^{-j90^\circ} = -j$ Volt

$I_{in}(t) = \cos(t)$ Ampere $\xrightarrow{phasor}$ $\bar{I}_{in} = 1$ Ampere

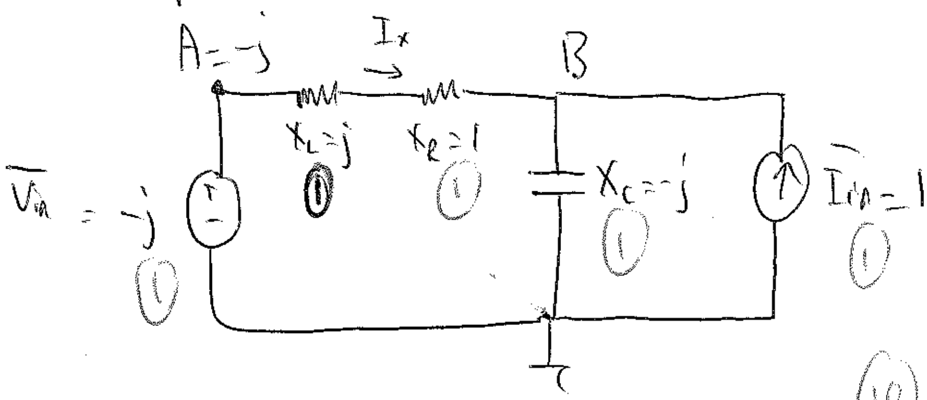


$X_L = j \cdot 1 \cdot 1 = j\ \Omega$

$X_C = \frac{1}{j\omega C} = \frac{1}{j} = -j\ \Omega$

$X_R = R = 1\ \Omega$

Both for $V_{in}(t)$ and $I_{in}(t)$ $\omega = 1\ \frac{rad}{sec}$



10

$$\frac{A - B}{X_L + X_R} + \bar{I}_{in} = \frac{B}{X_C} \qquad \frac{-j - B}{j + 1} + 1 = \frac{B}{-j} \qquad B = -j$$

2

$$I_x = \frac{A - B}{j + 1} = \frac{-j - (-j)}{j + 1} = 0 \quad 3$$

$$P_R = \frac{1}{2} I_x I_x^* R = \frac{1}{2} 0 \times 0 \times 1 = 0 \text{ Watt}$$

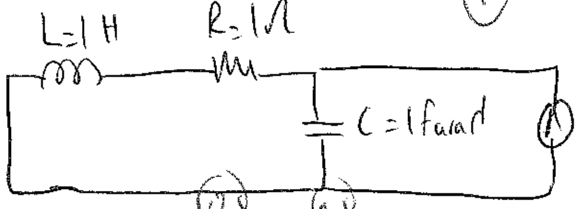
LILAS BELDEK

Q-2- $V_{in}(t) = \sin(t)$ Volt $\quad w_1 = 1 \frac{rad}{sec}$ $\qquad I_{in}(t) = \cos(2t)$ Ampere $\quad w_2 = 2 \frac{rad}{sec}$

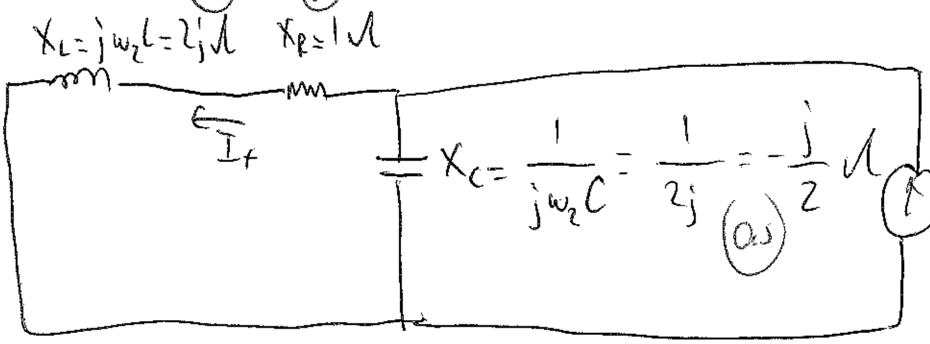
The angular frequencies of the sources are different

$$w_1 \neq w_2$$

Kill $V_{in}(t)$ (short circuit) $\qquad$ (1) → devre şekli



phaser



$$I_x = \bar{I}_{in} \times \frac{X_C}{X_L + X_R + X_C} = 1 \times \frac{-\frac{j}{2}}{2j+1-\frac{j}{2}} = \frac{-\frac{j}{2}}{1+\frac{3}{2}j} = \frac{-j}{2+3j} \quad (2-3j)$$

(4)

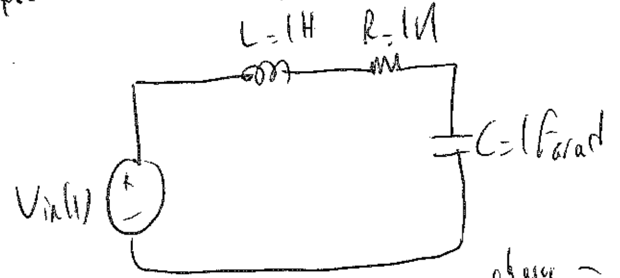
$$I_x = -\frac{(2+3j)}{13} \text{ Ampere}$$

(1)

$$P_{1,R} = \frac{1}{2} I_x I_x^{*} R = \frac{1}{2}(-1)\frac{(2+3j)}{13}(-1)\frac{(2-3j)}{13} \times 1 = \frac{13}{2 \times 13 \times 13} = \frac{1}{26} \text{ Watt}$$

(2)

Kapalı $I_{in}(t)$ (open circuit)



(1) → devre sabit

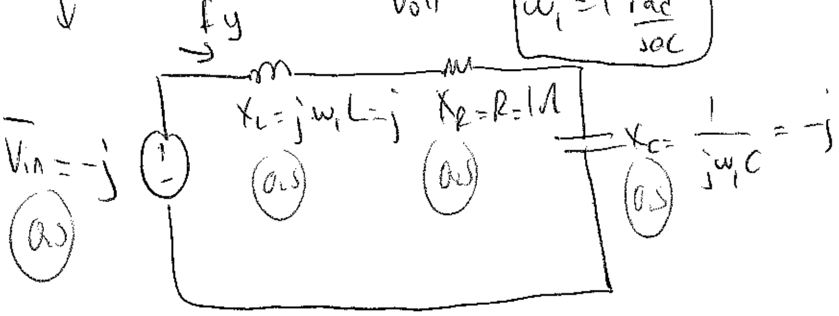
phasor

$V_{in}(t) = \sin(t)$ Volt $\xrightarrow{phasor}$ $\bar{V}_{in} = -j$ Volt

$\boxed{w_1 = 1\ \frac{rad}{sec}}$

$X_L = jw_1L = j\ \Omega$

$X_C = \frac{1}{jw_1C} = -j\ \Omega$



$$I_y = \frac{\bar{V}_{in}}{X_L + X_R + X_C} = \frac{-j}{j + (-j) + 1} = -j \quad (4)$$

(2)

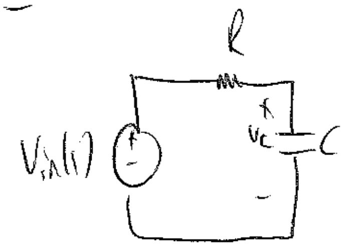
$$P_{2,R} = \frac{1}{2} I_y I_y^* R = \frac{1}{2} \cdot (-j)(j) \cdot 1 = \frac{1}{2} \text{ Watt}$$

$$P_R = P_{1,R} + P_{2,R} = \frac{1}{26} + \frac{1}{2} = \frac{14}{26} \text{ Watt (average power over } R)$$

(2)

Q-3 -

$R=1\,\Omega \quad C=1$ Farad



$$RC\frac{dV_c}{dt}+V_c=V_{in} \qquad \frac{dV_c}{dt}+V_c=V_{in}$$ (2)

1 Volt

$$\frac{dV_c}{dt}+V_c=\sin(t) \xrightarrow{\text{Laplace}} sV_c(s)-V_c(0)+V_c(s)=\frac{1}{s^2+1}$$ (5)

$$(s+1)V_c(s)=\frac{1}{s^2+1}+1 \qquad V_c(s)=\frac{1}{(s+1)(s^2+1)}+\frac{1}{s+1}$$ (8)

$$V_c(s)=\frac{A}{s+1}+\frac{Bs+C}{s^2+1}+\frac{1}{s+1}$$

$$\frac{A}{s+1}+\frac{Bs+C}{s^2+1}=\frac{1}{(s+1)(s^2+1)}$$

$(s^2+1) \quad (s+1)$ (5)

$$A(s^2+1)+(Bs+C)(s+1)=1$$

$$(A+B)s^2+(B+C)s+(A+C)=1$$

$$A=-B \qquad B=-C \qquad A+C=1$$

$$A=C=\frac{1}{2} \qquad B=-\frac{1}{2}$$

$$V_c(s)=\frac{1}{2}\frac{1}{s+1}-\frac{1}{2}\frac{s}{s^2+1}+\frac{1}{2}\frac{1}{s^2+1}+\frac{1}{s+1}$$ (8)

$\mathcal{L}^{-1}$

$$V_c(t)=\frac{1}{2}e^{-t}u(t)-\frac{1}{2}\cos(t)u(t)+\frac{1}{2}\sin(t)u(t)+e^{-t}u(t)$$ (2)

$$V_c(t)=\frac{3}{2}e^{-t}u(t)-\frac{1}{2}\cos(t)u(t)+\frac{1}{2}\sin(t)u(t)$$

$Q-4 \qquad H(s) = \dfrac{1000s}{(s+500)^2}$

(1) (a) $H(j\omega) = \dfrac{1000j\omega}{(j\omega+500)^2} = \dfrac{1000j\omega}{500^2-\omega^2+1000j\omega}$

(1) (b) $|H(j\omega)| = \dfrac{1000\omega}{\omega^2+500^2}$ or $H(j\omega) = \dfrac{1000\omega}{\sqrt{(500^2-\omega^2)^2+(1000\omega)^2}}$

(2) (d) $\angle H(j\omega) = 90 - 2\tan^{-1}\dfrac{\omega}{500}$

(2) (f) $|H(j\omega)|_{dB} = 20\log\dfrac{1000\omega}{\omega^2+500^2}$

(3) (g) At the resonant frequency $|H(j\omega)|$ is maximized $\left(\left.\dfrac{d|H(j\omega)|}{d\omega}\right|_{\omega_{res}} = 0\right)$

$$\frac{d}{d\omega}|H(j\omega)| = 1000\left[\frac{1(\omega^2+500^2) - 2\omega(\omega)}{(\omega^2+500^2)^2}\right]$$

$$= \frac{1000}{\omega^2+500^2}\left[500^2-\omega^2\right] \quad (2.5)$$

$$\left.\frac{d}{d\omega}|H(j\omega)|\right|_{\omega=\omega_{res}} = 0 \Rightarrow \frac{1000}{\omega_{res}^2+500^2}\left[500^2-\omega_{res}^2\right] = 0 \qquad \omega_{res} = 500 \ \frac{rad}{sec} \quad (0.5)$$

$$\left.|H(j\omega)|\right|_{\omega=\omega_{res}} = 1 = |H(j\omega)|_{max}$$

(5) (h) At corner frequencies $\left.|H(j\omega)|\right|_{\omega=\omega_c} = \dfrac{|H(j\omega)|_{max}}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$$\left.|H(j\omega)|\right|_{\omega=\omega_c} = \frac{1000\,\omega_c}{\omega_c^2+500^2} = \frac{1}{\sqrt{2}} \qquad \omega_c^2 - 1000\sqrt{2}\,\omega_c + 500^2 = 0$$

$$\Rightarrow \omega_c = \frac{1000\sqrt{2} \mp \sqrt{2\times10^6 - 4\times500^2}}{2} = \frac{1000\sqrt{2}\mp1000}{2} = 500\sqrt{2}\mp500 \ \frac{rad}{sec}$$

(10) (i) $|H(j\omega)|_{dB} = 20\log \dfrac{1000\,\omega}{\omega^2 + 500^2}$

$|H(j\omega)|_{dB} = 20\log 1000 + 20\log \omega - 20\log(\omega^2 + 500^2)$ (1)

if $0 < \omega \leq 500$ $\quad |H(j\omega)|_{dB} \cong 20\log 1000 + 20\log\omega - 20\log 500^2$

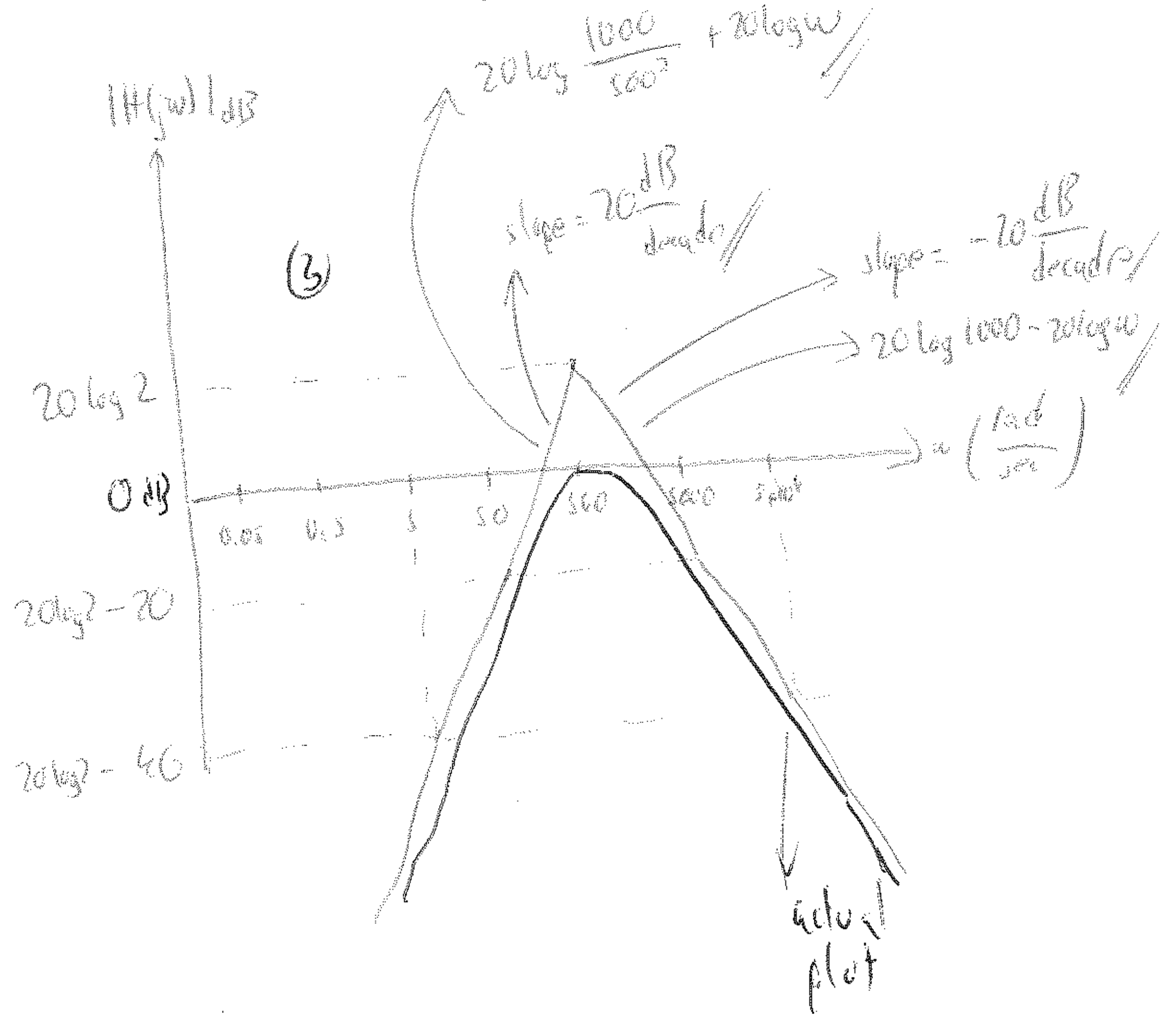
$= 20\log \dfrac{1000}{500^2} + 20\log\omega$ (2)

when $\omega = 500$ $\quad |H(j\omega)|_{dB} \cong 20\log 2$ (1)

if $500 \leq \omega$ $\quad |H(j\omega)|_{dB} \cong 20\log 1000 + 20\log\omega - 40\log\omega$

$= 20\log 1000 - 20\log\omega$ (2)

when $\omega = 500$ $\quad |H(j\omega)|_{dB} \cong 20\log 2$ (1)



(5) $\angle H(j\omega) = 90 - 2\,\mathrm{Tan}^{-1}\dfrac{\omega}{500}$

(14)

(3) $0 < \omega < \dfrac{500}{10} \Rightarrow \angle H(j\omega) = 90^\circ$

(3) $\dfrac{500}{10} < \omega < 10 * 500 \Rightarrow \angle H(j\omega) = \left[90 - 90 \log \dfrac{\omega}{50}\right]^\circ$ (linear interpolation in logarithmic scale)

(3) $10 * 500 < \omega \Rightarrow \angle H(j\omega) = 90 - 2 * 90 = -90^\circ$

(5)